Indirect Methods and Halo Nuclei

International Graduate School Basel-Graz-Tübingen

At the Interface of Particle, Nuclear and Atomic Physics

in Honor of Gerhard Baur and Dirk Trautmann

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Excellence Cluster 'Universe'

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Outline

- **Part I: Motivation**
  nuclear reactions of astrophysical interest, reaction rates and cross sections, charged-particle reactions, halo nuclei
- **Part II: Halo Nuclei**
  characteristic parameters, ground state properties, electromagentic excitation, radial integrals, wave functions, transition strength and shape functions
- **Part III: Indirect Methods**
  - overview, transfer reactions, T matrix elements, spectroscopic factors
  - Coulomb dissociation: theory, parameters, cross sections, higher-order effects
  - ANC method: theory, continuum interaction
  - Trojan-Horse method: theory, application, electron screening
- **Summary**

recurrent themes: asymptotics of wave functions, factorization

Part I:

Motivation

Nuclear Reactions of Astophysical Interest

nuclear astrophysics

- nuclear reaction rates are basic input in many astrophysical models
  (primordial nucleosynthesis, stellar evolution, novae, supernovae, . . .)
  for various processes (pp chains, CNO cycles, s-, r-, p-, rp-process, . . .)
- ideally: direct measurement of reaction cross sections at relevant energies
  but in most cases practically impossible (small cross sections, often unstable nuclei)
- alternative approaches ?
  depend on type of reaction

Types of Reactions

- radiative capture/
  photo dissociation reactions:

  (n,γ), (p,γ), (α,γ), ... /
  (γ,n), (γ,p), (γ,α), ...

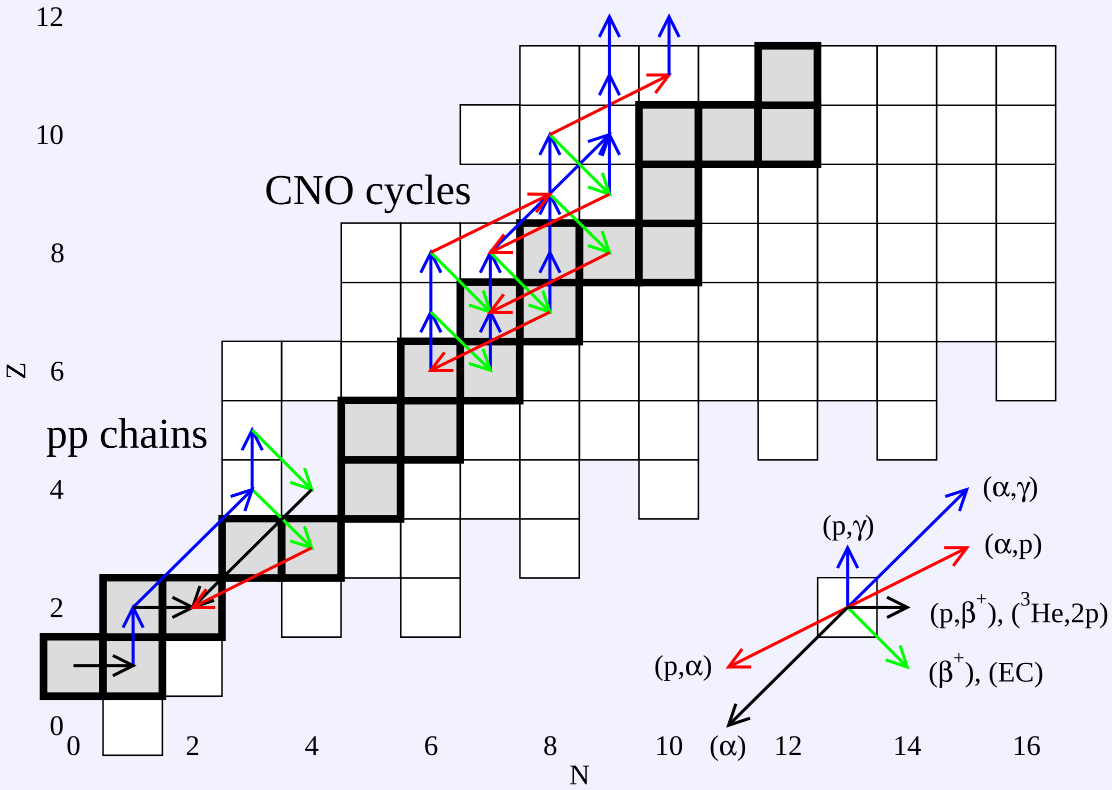
- nuclear rearrangement reactions:

  (p,α), (α,p), $(^3\text{He},2p)$, ...

- weak interaction reactions:

  β^+, β^-, electron capture (EC)

- ...



here:

- only charged-particle reactions
- only reactions with electromagnetic or strong interaction

Reaction Rates and Cross Sections

astrophysical environment

nuclei in hot plasma

⇒ temperature-dependent distribution of relative velocities v for reaction $b + c \to \ldots$

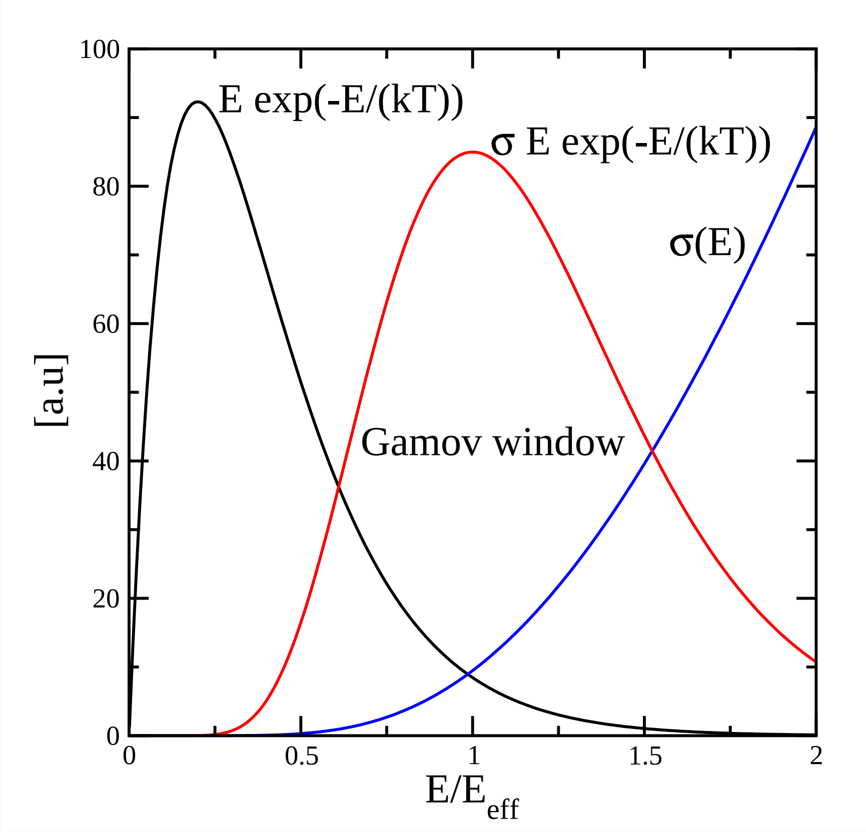
⇒ relevant quantity:

Maxwellian-averaged reaction rate

$$r_{bc} = \frac{\varrho_b \varrho_c}{1+\delta_{bc}} \langle \sigma v \rangle$$

with densities ϱ_b, ϱ_c and

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu_{bc}}} \int_0^\infty \sigma(E)\ E\ e^{-\frac{E}{kT}} \frac{dE}{(kT)^{3/2}}$$



⇒ cross sections σ needed in Gamov window of width ΔE around effective energy E_{eff}

Gamov Window

parameters

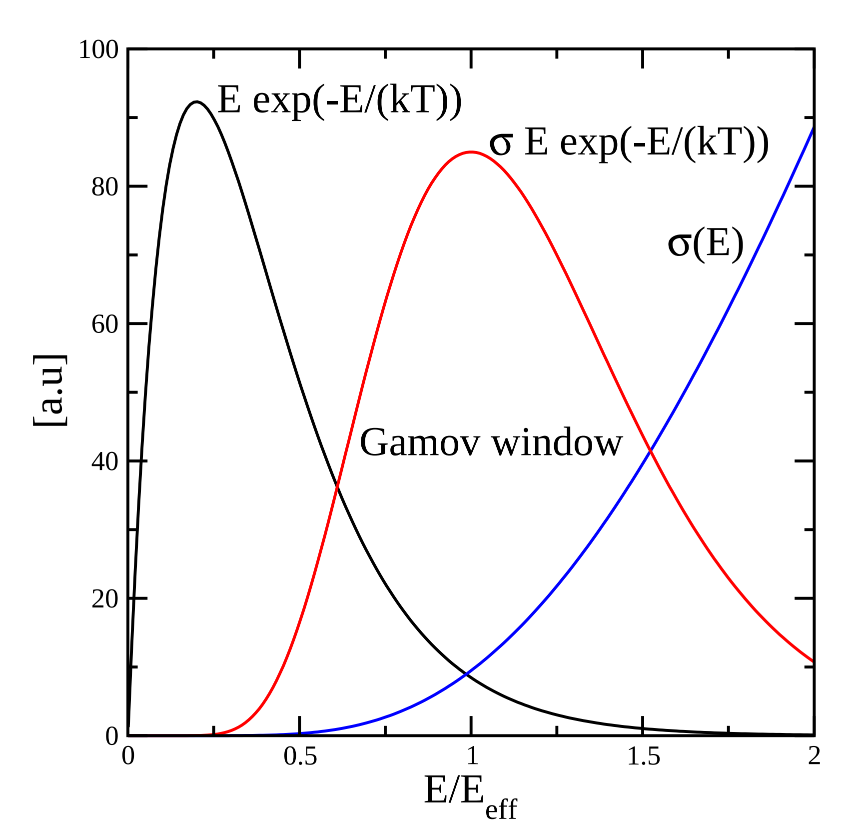
- effective energy

$$E_{\text{eff}} = 0.1220\, \mu_{bc}^{1/3} (Z_b Z_c T_9)^{2/3} \text{ MeV}$$

- width

$$\Delta E = 0.2368\, \mu_{bc}^{1/6} (Z_b Z_c)^{1/3} T_9^{5/6} \text{ MeV}$$

with temperature T_9 in 10^9 K
and reduced mass $\mu_{bc} = \frac{m_b m_c}{m_b + m_c}$ in amu



reaction	E_{eff} [keV]	$\sigma(E_{\text{eff}})$ [pb]
^{3}He(^{3}He,2p)^{4}He	22.0	1.5
^{7}Be(p,γ)^{8}B	18.4	1.5×10^{-3}
^{3}He(α,γ)^{7}Be	23.0	3.0×10^{-5}
^{14}N(p,γ)^{15}O	27.2	2.2×10^{-7}

for $T = 15.5 \times 10^6$ K (center of the sun)

Charged-Particle Reactions

- **Coulomb barrier** in reaction $b + c \to \ldots$ with charged nuclei b, c

  ⇒ extremely small cross sections $\sigma(E)$ with strong energy dependence

  ⇒ astrophysical relevant energies (Gamov window) usually not accessible

  ⇒ measurement at higher energies and extrapolation to low energies E

  with help of astrophysical S factor $S(E) = \sigma(E) E \exp(2\pi\eta)$

  Sommerfeld parameter $\eta = Z_b Z_c e^2/(\hbar v)$

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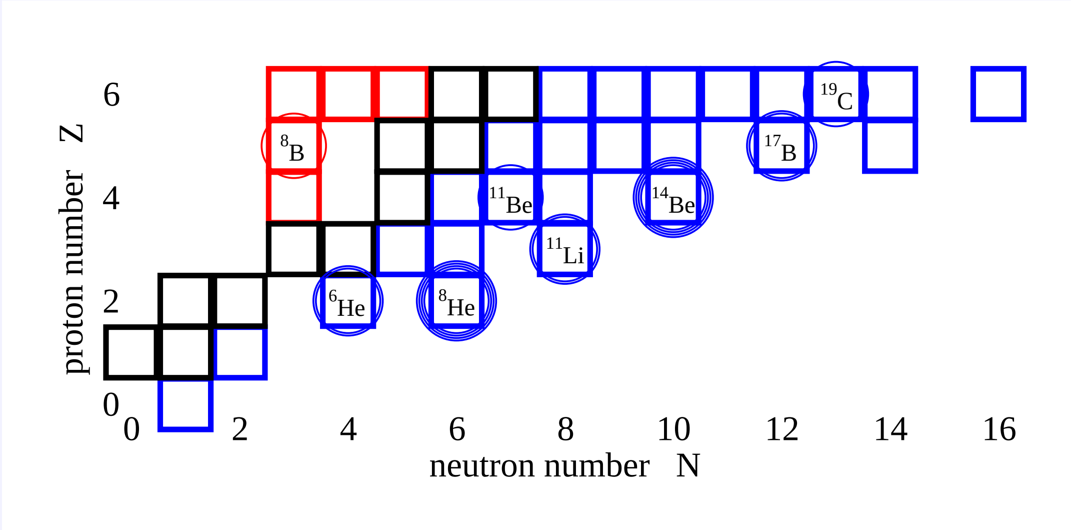
- direct measurement very difficult, often unstable nuclei involved

$\Rightarrow$ indirect methods

Halo Nuclei

Light exotic nuclei:



- large neutron/proton excess
- radioactive with short lifetimes

close to neutron/proton driplines

⇒ halo phenomenon observed

- large size (i.e. interaction radius)
- large cross sections for electromagnetic excitation to low energies in the continuum
- breakup rections: narrow relative-momentum distributions of fragments

⇒ extraction of spectroscopic information

- small separation energy of last nucleon
- ground state well described in single-particle picture (nucleon + core)

essential features characterized by few **low-energy constants** ⇒ **scaling laws**

Part II:

Halo Nuclei

Halo Nuclei - Characteristic Parameters

halo nucleus ($a = b + c$) with **nucleon** (b) + **core** (c) structure
with effective mass $\mu = \frac{m_b m_c}{m_b + m_c}$ and charge numbers Z_b and Z_c

- relevant scales of the system

origin	energy	momentum	dimensionless quantity
ground state	one-nucleon separation energy S_b	$\hbar q = \sqrt{2\mu S_b}$ inverse decay length q	$\gamma = qR$
scattering state	relative energy E in continuum	$\hbar k = \sqrt{2\mu E}$ relative momentum	$\kappa = kR$
Coulomb field	Gamov energy $E_G = \left(\frac{Z_b Z_c e^2}{\hbar}\right)^2 \frac{\mu}{2}$	$\hbar / a_B$ with nuclear Bohr radius $a_B = \hbar / \sqrt{2\mu E_G}$	$\eta_i = \sqrt{\frac{E_G}{S_b}} = \frac{1}{q a_B}$ $\eta_f = \sqrt{\frac{E_G}{E}} = \frac{1}{k a_B}$

with radius $R \Leftrightarrow$ typical size of the system

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with radius $R \Leftrightarrow$ typical size of the system

- three independent dimensionless quantities $\Rightarrow$ relevant for systematic expansions

$$\gamma = qR \qquad x = \frac{\kappa}{\gamma} = \sqrt{\frac{E}{S_b}} \qquad \eta_i = \frac{1}{q a_B} = x \eta_f$$

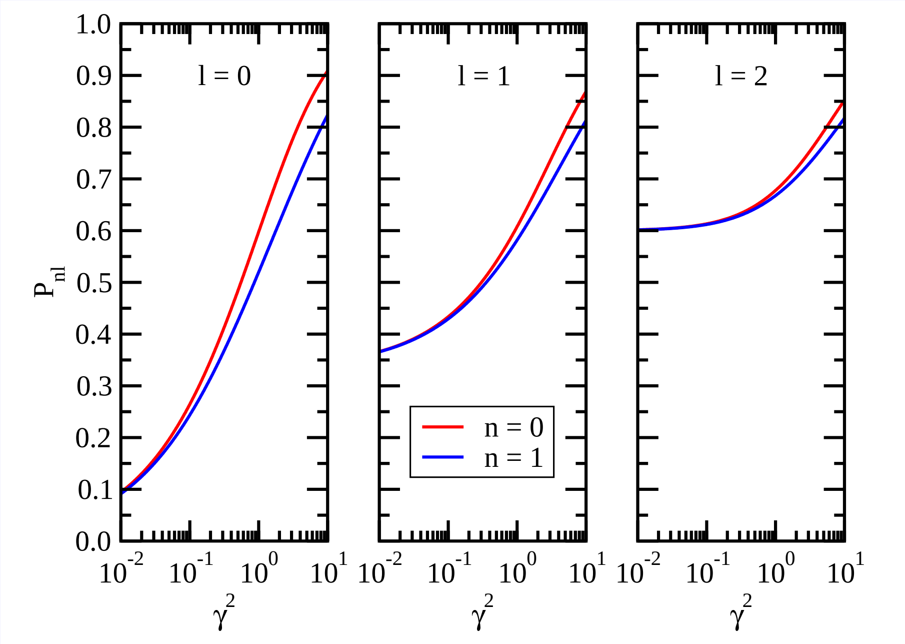
($\eta_i = \eta_f = 0$ for neutron + core systems)

(S. Typel and G. Baur, Nucl. Phys. A 759 (2005) 247)

Neutron Halo Nuclei - Ground State Properties

square-well potential of radius R and separation energy S_n, i.e. $\gamma^2 = 2\mu S_n R^2/\hbar^2$

- probability P_{nl} to find the neutron
  inside potential well (n = number of nodes)
  in the limit $\gamma \longrightarrow 0$:
  – $P_{n0} \longrightarrow 0$ for $l = 0$
  – $P_{nl} \longrightarrow (2l-1)/(2l+1)$ for $l \geq 1$



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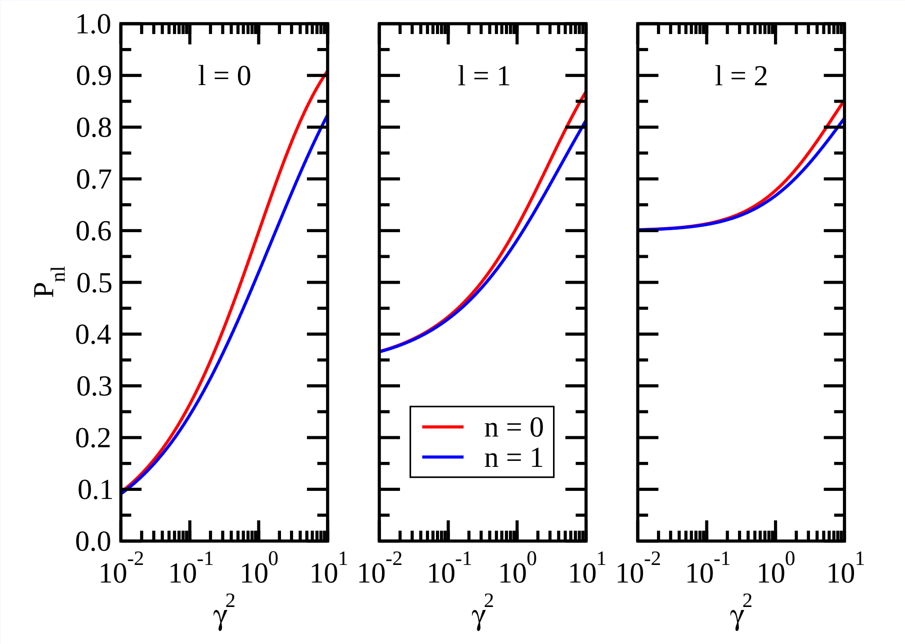
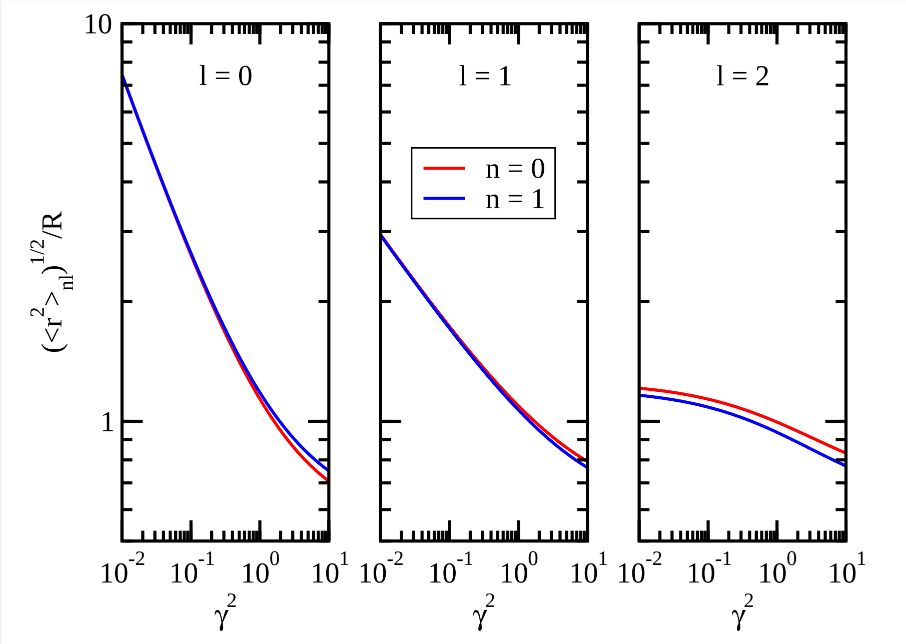
  - $P_{n0} \to 0$ for $l = 0$
  - $P_{nl} \to (2l-1)/(2l+1)$ for $l \geq 1$
- rms radius $(\langle r^2 \rangle_{nl})^{1/2}$ in the limit $\gamma \to 0$
  - $\langle r^2 \rangle_{nl} \propto \gamma^{-2}$ for $l = 0$
  - $\langle r^2 \rangle_{nl} \propto \gamma^{-1}$ for $l = 1$

  $\Rightarrow \langle r^2 \rangle_{nl}$ diverges for s and p waves
  - $\langle r^2 \rangle_{nl}$ finite for $l \geq 2$





halo effect exists only for low orbital angular momenta and small binding energies

Electromagnetic Excitation

electromagnetic transition of type $\pi = E, M$ and multipolarity $\lambda = 1, \ldots$

- photo dissociation cross section

$$\sigma_{\pi\lambda}(a + \gamma \to b + c) = \frac{\lambda + 1}{\lambda} \frac{(2\pi)^3}{[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1} \frac{dB(\pi\lambda)}{dE}$$

- reduced transition probability

$$\frac{dB}{dE}(\pi\lambda, J_i \to kJ_f) = \frac{2J_f + 1}{2J_i + 1} \sum_{j_f l_f} \left| \sum_{j_i l_i j_c} \langle kJ_f j_f l_f s j_c || \mathcal{M}(\pi\lambda) || J_i j_i l_i s j_c \rangle \right|^2 \frac{\mu k}{(2\pi)^3 \hbar^2}$$

- reduced matrix element $\langle kJ_f j_f l_f s j_c || \mathcal{M}(\pi\lambda) || J_i j_i l_i s j_c \rangle$

- electric transition $\Rightarrow$ multipole operator $\mathcal{M}(E\lambda\mu) = Z_{\text{eff}}^{(\lambda)} e\, r^\lambda Y_{\lambda\mu}(\hat{r})$

  with effective charge number $Z_{\text{eff}}^{(\lambda)} = Z_b \left(\frac{m_c}{m_b+m_c}\right)^\lambda + Z_c \left(-\frac{m_b}{m_b+m_c}\right)^\lambda$

Radial Integrals

- **example**: breakup of ^{11}Be $\rightarrow$ ^{10}Be $+ n$
  neutron halo nucleus with neutron separation energy $S_n = 0.504$ MeV
  $E1$ transition from s wave bound state to p wave scattering state with energy E

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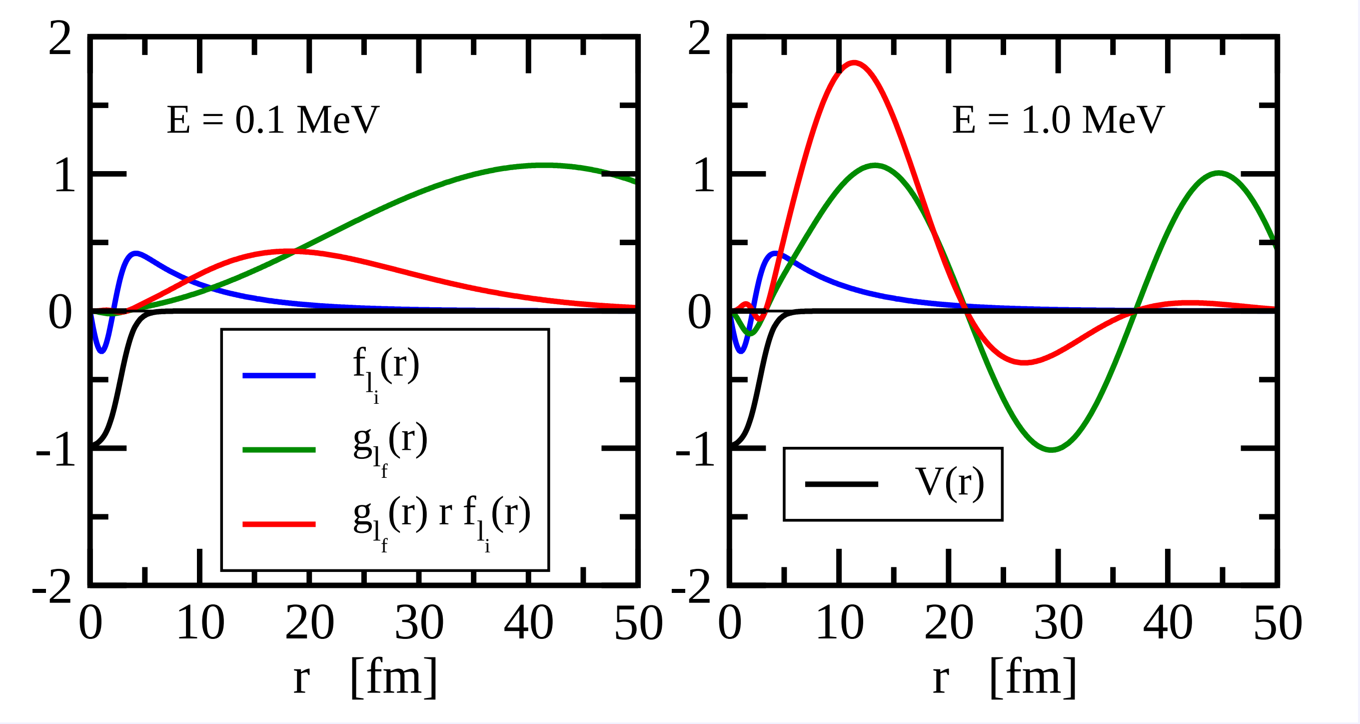

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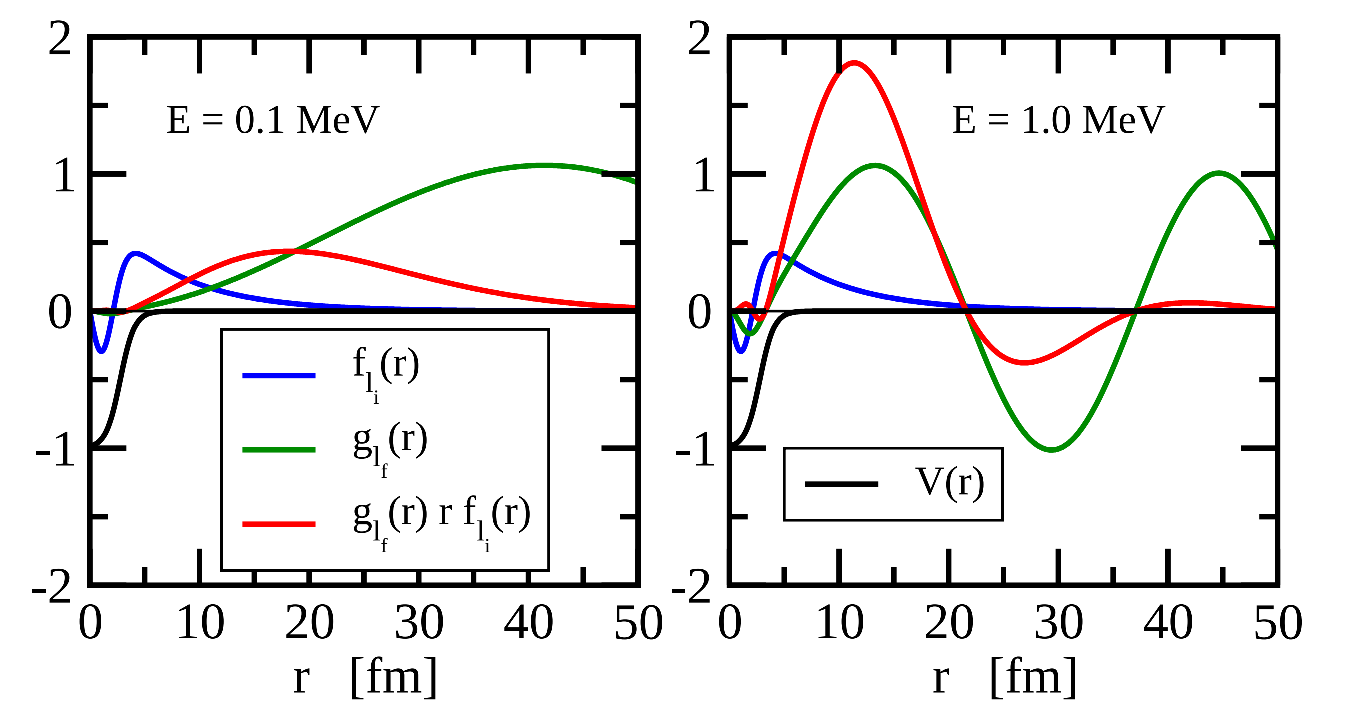

- Eλ transitions at low relative energies
  $\Rightarrow$ matrix elements determined by asymptotic of wave functions

Wave Functions - General

relative motion of two-body system $a = b + x \, (= c + y = d + z = \ldots)$

- many-body wave function Ψ_a is solution of Schrödinger equation

  $$H\Psi_a = (T + V_a)\,\Psi_a = E\Psi_a$$

  with potential $V_a = V_{bx}^C + V_{bx}^N = V_{cy}^C + V_{cy}^N$ (Coulomb + nuclear interaction)
  and boundary condition for bound/scattering state

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  and boundary condition for bound/scattering state

- for large distances $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, etc.:

  Coulomb interaction remains, short-range nuclear interaction vanishes

  $\Rightarrow$ universal asymptotic form of $\Psi_a \to \phi_b \phi_x \psi_{bx}(r_{bx}) + \ldots$

  for $r_{bx}, r_{cy}, \ldots \to \infty$ with relative wave functions $\psi_{bx}, \psi_{cy}, \ldots$

- exact solution: partial wave expansion

Wave Functions - Bound States

general form of asymptotics (without particle spins, $\alpha = (bx), (cy), \ldots$)

$$\psi_\alpha(m) \to \frac{1}{r_\alpha} \sum_l f_{\alpha l}(r_\alpha) Y_{lm}(\hat{r}_\alpha)$$ for $r_\alpha \to \infty$

with radial wave functions $f_{\alpha l}(r_\alpha) = C_\alpha^a(l) W_{-\eta_\alpha, l+1/2}(2q_\alpha r_\alpha)$

and angular parts $Y_{lm}(\hat{r}_\alpha)$ (spherical harmonic)

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- Whittaker function $W_{-\eta_\alpha, l+1/2}(2q_\alpha r_\alpha) \to \exp(-q_\alpha r_\alpha)$
  with Sommerfeld parameter η_α, bound-state parameter q_α
  e.g. $\eta_{bx} = \frac{Z_b Z_x e^2 \mu_{bx}}{\hbar^2 q_{bx}} \qquad q_{bx} = \sqrt{2\mu_{bx} S_{bx}}/\hbar$
  and separation energy S_{bx} of particle a into b and x
- asymptotic normalization coefficient (ANC) $C_\alpha^a(l)$

Wave Functions - Scattering States

general form of asymptotics (without particle spins)

$$\Psi_{bx}^{(+)} \to \frac{4\pi}{k_{bx}} \sum_{\alpha=(bx),(cy),\ldots} \phi_\alpha \frac{1}{r_\alpha} \sqrt{\frac{v_{bx}}{v_\alpha}} \sum_{lm} g_{\alpha l}^{(+)}(r_\alpha) i^l Y_{lm}(\hat{r}_\alpha) Y_{lm}^*(\hat{k}_{bx})$$ for $r_\alpha \to \infty$

with radial wave functions $g_{\alpha l}^{(+)}(r_\alpha) = \frac{1}{2i}\left[S_{\alpha(bx)}^{l} u_l^{(+)}(\eta_\alpha, k_\alpha r_\alpha) - \delta_{\alpha(bx)} u_l^{(-)}(\eta_\alpha, k_\alpha r_\alpha)\right]$

and angular parts $Y_{lm}(\hat{r}_\alpha)$, $Y_{lm}(\hat{k}_\alpha)$ (spherical harmonics)

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and angular parts $Y_{lm}(\hat{r}_\alpha)$, $Y_{lm}(\hat{k}_\alpha)$ (spherical harmonics)

- Coulomb wave functions
  $u_l^{(\pm)}(\eta_\alpha, k_\alpha r_\alpha) = e^{\mp i\sigma_l} \left[G_l \pm i F_l \right] \to \exp\left\{ \pm i \left[k_\alpha r_\alpha - 2\eta_\alpha \ln(k_\alpha r_\alpha) - \frac{l\pi}{2} \right] \right\}$
  with Sommerfeld parameter η_α, momentum $\hbar \vec{k}_\alpha$, energy E_{bx} of relative motion
  e.g. $\eta_{bx} = \frac{Z_b Z_x e^2 \mu_{bx}}{\hbar^2 k_{bx}} \qquad k_{bx} = \sqrt{2\mu_{bx} E_{bx}}/\hbar \qquad \mu_{bx} = \frac{m_b m_x}{m_b + m_x}$

- S-matrix elements $S_{\alpha(bx)}^l$
  e.g. elastic scattering $S_{\alpha\alpha}^l = e^{2i[\sigma_l + \delta_l(\alpha)]}$ with Coulomb and nuclear phase shifts

Reduced Transition Probability and Shape Functions

- reduced transition probability for $E\lambda$ transition $l_i \to l_f$:

$$\frac{dB(E\lambda)}{dE} = \left[Z_{\rm eff}^{(\lambda)} e\right]^2 \frac{2\mu}{\pi\hbar^2} D_s \frac{|C_{l_i}|^2}{q^{2\lambda+3}} \mathcal{S}_{l_i}^{l_f}(\lambda)$$ with spin factor D_s

  - scaling of transition strength with q through factor $|C_{l_i}|^2/q^{2\lambda+3}$ with ANC
  - scaling of C_{l_i} for neutron + core systems: $C_0 \propto \sqrt{q}$, $C_{l_i} \propto q^{l_i}$ for $l_i \geq 1$

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  - scaling of C_{l_i} for neutron + core systems: $C_0 \propto \sqrt{q}$, $C_{l_i} \propto q^{l_i}$ for $l_i \geq 1$

- dimensionless shape function $\mathcal{S}_{l_i}^{l_f}(\lambda) = \frac{q}{k} \left| \mathcal{I}_{l_i}^{l_f}(\lambda) \right|^2$

- dimensionless reduced radial integral

$$\mathcal{I}_{l_i}^{l_f}(\lambda) = q^{\lambda+1} \int_R^\infty dr\, r^\lambda \left[\cos(\delta_{l_f}) F_{l_f}(kr) + \sin(\delta_{l_f}) G_{l_f}(kr)\right] W_{-\eta_i, l_i+1/2}(2qr)$$

  - depends only on γ, x, η_i and phase shift δ_{l_f}
  - analytical for neutron + core systems

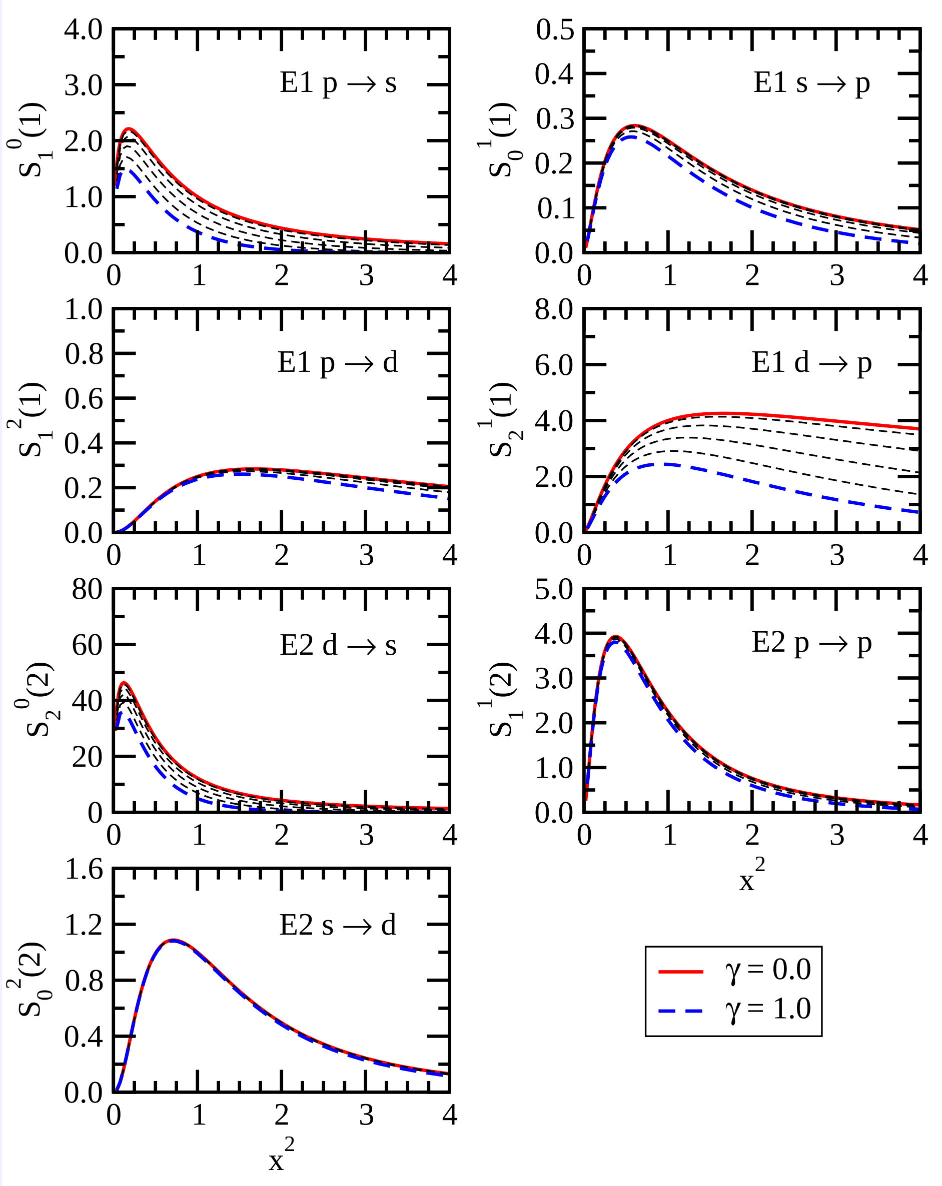
(S. Typel and G. Baur, Nucl. Phys. A 759 (2005) 247)

Shape Functions for Neutron + Core Systems I

- without final-state interaction ($\delta_{l_f} = 0$)
- characteristic shape of $\mathcal{S}_{l_i}^{l_f}(\lambda)$
  depending on $E\lambda$ transition $l_i \to l_f$
- strong peak at small $x^2 = E_{\rm rel}/S_n$
- hierarchy of maxima
  (shift to larger x^2 with increasing l_f)
- variation with γ
  (smaller sensitivity with larger l_f)
- analytical forms for $\gamma = 0$
  (extreme halo limit) e.g.

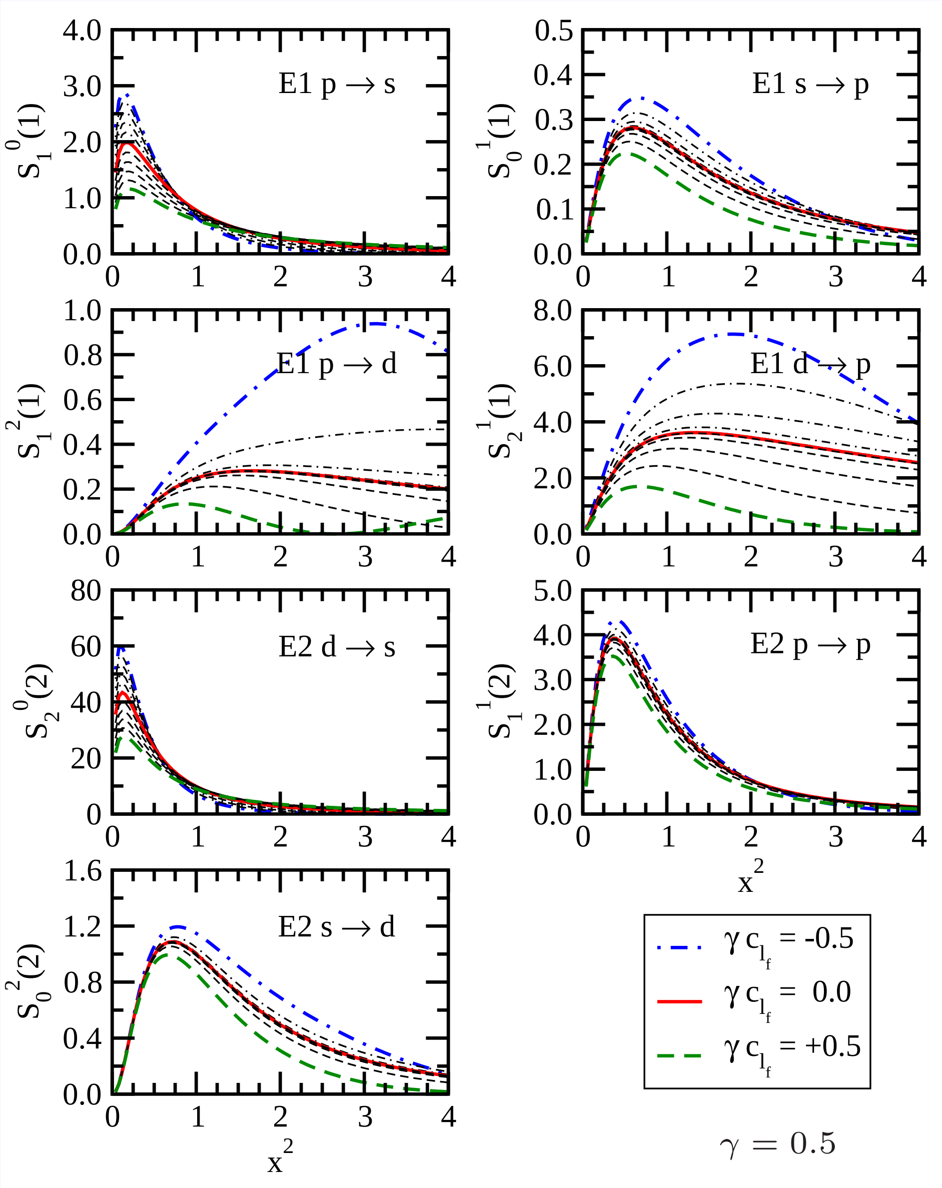
$$\mathcal{S}_1^0(1) = \frac{x(3+x^2)^2}{(1+x^2)^4}$$

$$\mathcal{S}_0^1(1) = \frac{4x^3}{(1+x^2)^4}$$



Shape Functions for Neutron + Core Systems II

- with final-state interaction ($\delta_{l_f} \neq 0$)
  use effective-range expansion
  $$\tan(\delta_{l_f}) = -(x\gamma c_{l_f})^{2l_f+1}$$
  with constant scattering length
  $$a_{l_f} = (c_{l_f} R)^{2l_f+1}$$
- "natural" values: $\gamma c_{l_f} \leq 1$
  (zero-range potential: $a_0 = 1/q$ exact)
- expansion of analytical forms for small γ:
  $$\mathcal{S}_1^0(1) = \frac{x(3+x^2)^2}{(1+x^2)^4}\left[1 - \frac{4}{3+x^2}\gamma c_0 + \ldots\right]$$
  $$\mathcal{S}_0^1(1) = \frac{4x^3}{(1+x^2)^4}\left[1 - (1+3x^2)c_1^3\gamma^3 + \ldots\right]$$
- effect of final-state interaction
  even for $x \to 0$



Shape Functions for Proton + Core Systems

- without final-state interaction $(\delta_{l_f} = 0)$
- halo limit $\gamma \to 0$
- increase of Sommerfeld parameter η_i

  $\Downarrow$

  – shift of maximum to larger $x^2 = \frac{E_{\mathrm{rel}}}{S_p}$

  – broadening of peak

  – reduction of strength

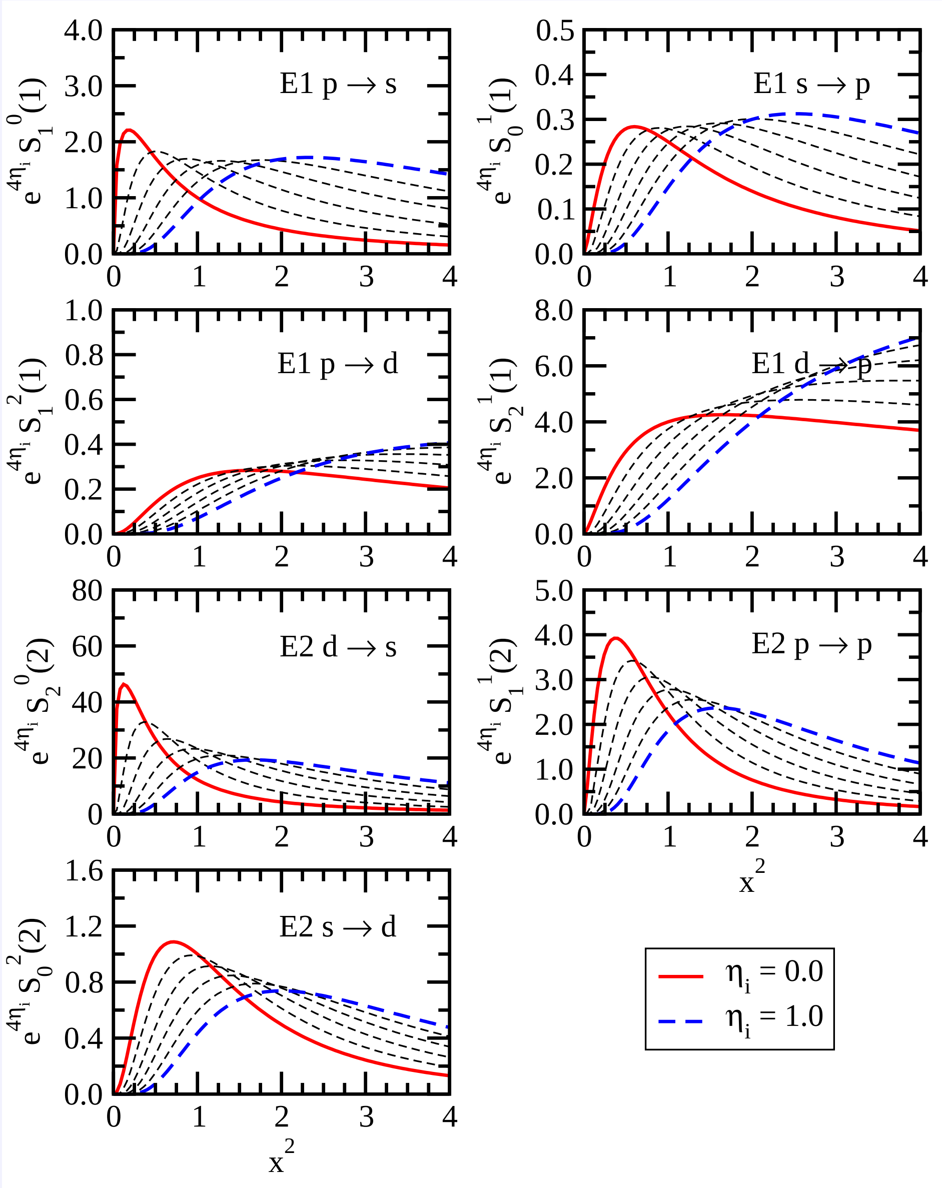
  (scaling of $\mathcal{S}_{l_i}^{l_f}(\lambda)$ with $\exp(4\eta_i)$)

  – less sensitivity to l_i, l_f

  (Coulomb barrier dominates)

- highly charged core (= large nuclei)

  $\Rightarrow$ no halo effect



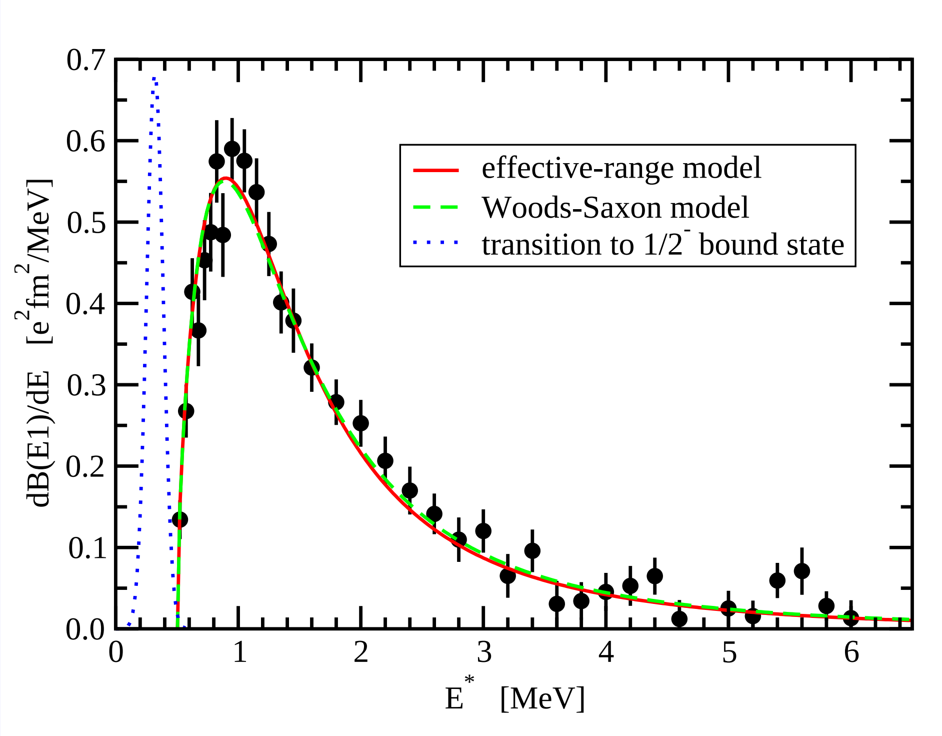
Coulomb Dissociation of ^{11}Be

- $E1$ transition from s-wave halo ground state ($S_n = 504$ keV, $\gamma = 0.41$, $R = 2.78$ fm) to p-wave continuum states with $j = 3/2, 1/2$
- effective-range expansion for phase shifts

$$\tan \delta_l^j = -(c_l^j x \gamma)^{2l+1}$$

  with reduced scattering length c_l^j
- fit to experimental data from Coulomb breakup of ^{11}Be at 520 A·MeV on Pb
  $\Rightarrow$ ANC $C_0 = 0.724(8)$ fm$^{-1/2}$
  $\Rightarrow$ spectroscopic factor $C^2 S = 0.704(15)$
  $\Rightarrow$ reduced scattering lengths
  $c_1^{3/2} = -0.41(86, -20)$
  $c_1^{1/2} = 2.77(13, -14)$

(S. Typel and G. Baur, Phys. Rev. Lett. 93 (2004) 142502)



exp. data: R. Palit et al., PRC 68 (2003) 034318

- $c_1^{1/2}$ unnaturally large
  $\Leftrightarrow$ existence of bound $1/2^-$ state 320 keV above ground state
  $\Rightarrow$ reduced $E1$ strength in continuum

Part III:

Indirect Methods

Indirect Methods - Overview I

Coulomb dissociation

G. Baur et al.,
NPA 458 (1986) 188

- study inverse of radiative capture reaction
  $b(x,\gamma)a \;\Leftrightarrow\; a(\gamma,x)b$
- use Coulomb field of target nucleus A as source of photons
  $a(\gamma,x)b \;\Leftrightarrow\; A(a,bx)A$

$\Downarrow$

absolute S factors
as a function of energy

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ANC method
H. M. Xu et al.,
PRL 73 (1994) 2027

- extract asymptotic normalization coefficient of ground state wave function of nucleus a from transfer reactions
- calculate matrix elements for radiative capture reaction $b(x,\gamma)a$

$\Downarrow$

S factor at zero energy

Indirect Methods - Overview I

Coulomb dissociation
G. Baur et al.,
NPA 458 (1986) 188

- study inverse of radiative capture reaction
  $b(x,\gamma)a \;\Leftrightarrow\; a(\gamma,x)b$
- use Coulomb field of target nucleus A as source of photons
  $a(\gamma,x)b \;\Leftrightarrow\; A(a,bx)A$

$\Downarrow$

absolute S factors
as a function of energy

ANC method
H. M. Xu et al.,
PRL 73 (1994) 2027

- extract asymptotic normalization coefficient of ground state wave function of nucleus a from transfer reactions
- calculate matrix elements for radiative capture reaction $b(x,\gamma)a$

$\Downarrow$

S factor at zero energy

Trojan-Horse method
G. Baur,
PLB 178 (1986) 35

- study three-body reaction
  $A+a \to C+c+b$
  with Trojan horse
  $a = b + x$
  and spectator b
- extract cross section of two-body reaction
  $A + x \to C + c$

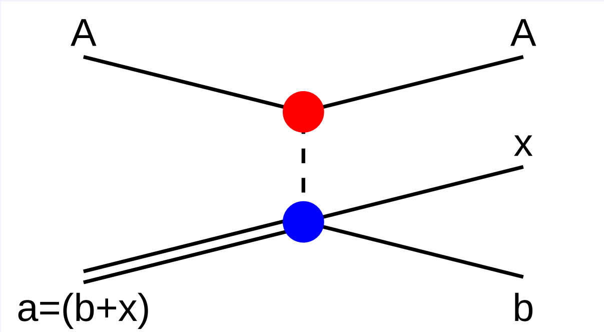
$\Downarrow$

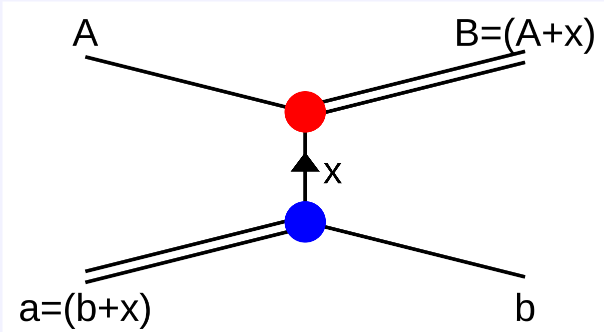
energy dependence
of S factor

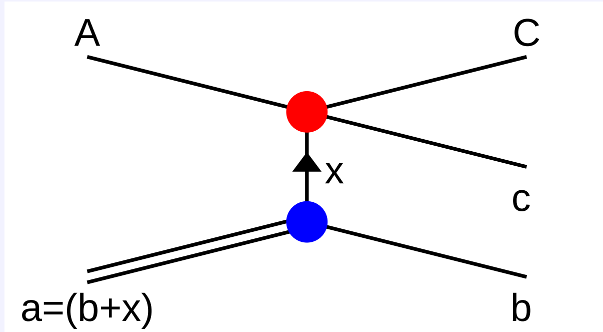
theoretical description? relation of methods?

Indirect Methods - Overview II

Coulomb dissociation | **ANC method** | **Trojan-Horse method**







photon exchange

transfer of particle to bound state

transfer of particle to continuum state

- similar reaction mechanisms: transfer of virtual particle
- final state with three particles (bound/continuum states)
- theoretical descripton with direct reaction theory

Indirect Methods - Overview III

general characteristics:

- two-body reaction at low-energy is replaced by three-body reaction at "high-energy" with large cross section
  - Coulomb dissociation $b(x,\gamma)a \Rightarrow A(a,bx)A$
  - ANC method $b(x,\gamma)a \Rightarrow A(a,B)b$ $\quad a = (b+x) \quad B = (A+x)$
  - Trojan-horse method $A(x,c)C \Rightarrow A(a,Cc)b$

Indirect Methods - Overview III

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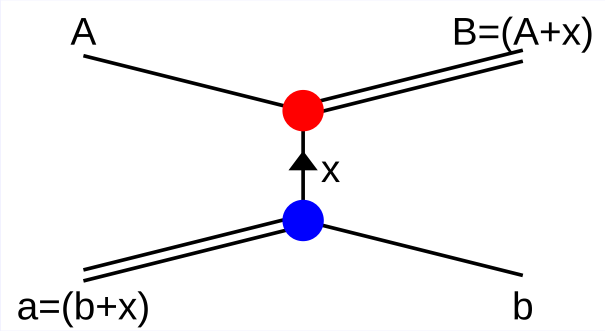
  - ◦ Trojan-horse method $A(x,c)C \Rightarrow A(a,Cc)b$
- transfer of virtual particle (photon γ or nucleus x)
- relation of cross sections is found with the help of nuclear direct reaction theory
- theoretical approximations essential
- study of peripheral reactions
  - – asymptotics of wave functions relevant
  - – selection of suitable kinematical conditions important

Transfer Reactions - Cross Sections

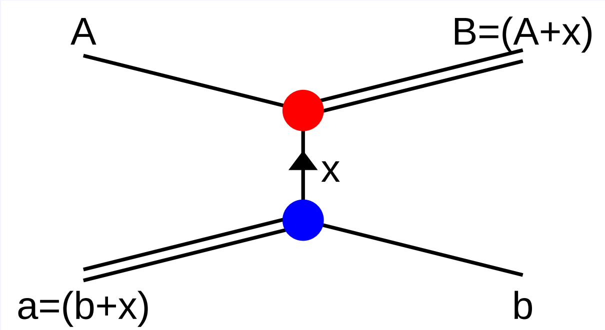
- **transfer reaction** $A + a \longrightarrow B + b$ with $a = b + x$
  to bound state of $B = A + x$



- **cross section**

$$d\sigma = \frac{2\pi}{\hbar}\frac{\mu_{Aa}}{p_{Aa}}\left|T_{fi}\right|^2 \delta(E_B + E_b - E_A - E_a - Q)\frac{d^3 p_{Bb}}{(2\pi\hbar)^3}$$

Transfer Reactions - Cross Sections



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with T-matrix in prior formulation $T_{fi} = \langle \Psi_{Bb}^{(-)} | V_{Aa}^{(i)} | \exp(i\vec{k}_{Aa} \cdot \vec{r}_{Aa}) \phi_A \phi_a \rangle$

or in post formulation $T_{fi} = \langle \exp(i\vec{k}_{Bb} \cdot \vec{r}_{Bb}) \phi_B \phi_b | V_{Bb}^{(f)} | \Psi_{Aa}^{(+)} \rangle$

$\Psi_{Aa}^{(+)} / \Psi_{Bb}^{(-)}$: exact scattering wave functions in initial/final state

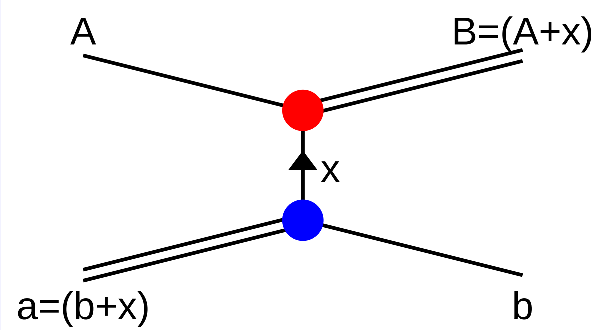
$V_{Aa}^{(i)} / V_{Bb}^{(f)}$: full potentials in initial/final state

- T-matrix element contains essential information on reaction process
- transferred particle x is virtual, i.e. $E_x \neq \frac{p_x^2}{2m_x}$
- two poles in diagram $\Rightarrow$ factorization

T-Matrix Elements - Transformations

- introduce optical potentials U_{ij} $(ij = Aa, Bb)$ and

  distorted waves $\chi_{ij}^{(\pm)}$ with $(\hat{T}_{ij} + U_{ij})\chi_{ij}^{(\pm)} = E_{ij}\chi_{ij}^{(\pm)}$



- apply Gell-Mann–Goldberger relation
  (Phys. Rev. 91 (1953) 398) $\Rightarrow$
  - prior form $T_{fi} = \langle \Psi_{Bb}^{(-)} | V_{Aa}^{(i)} - U_{Aa} | \chi_{Aa}^{(+)} \phi_A \phi_a \rangle$ exact!
  - post form $T_{fi} = \langle \chi_{Bb}^{(-)} \phi_B \phi_b | V_{Bb}^{(f)} - U_{Bb} | \Psi_{Aa}^{(+)} \rangle$ exact!

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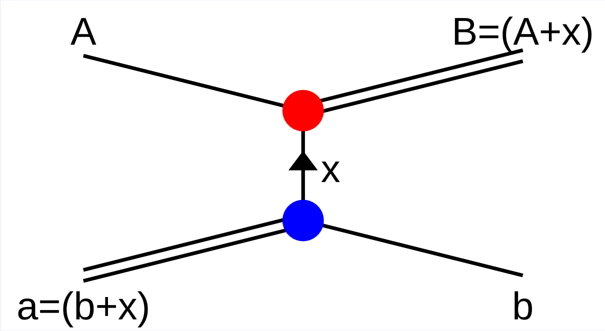

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  - post form $T_{fi} = \langle \chi_{Bb}^{(-)} \phi_B \phi_b | V_{Bb}^{(f)} - U_{Bb} | \Psi_{Aa}^{(+)} \rangle$ exact!

- distorted-wave Born approximation (DWBA):

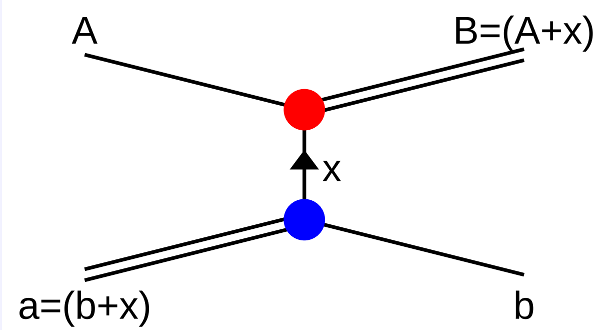
  replace exact scattering wave functions $\Psi_{Aa}^{(+)} \to \chi_{Aa}^{(+)} \phi_A \phi_a$ or $\Psi_{Bb}^{(-)} \to \chi_{Bb}^{(-)} \phi_B \phi_b$

  - prior form $T_{fi} \approx \langle \chi_{Bb}^{(-)} \Phi_{Ax}^{B} | V_{Aa}^{(i)} - U_{Aa} | \chi_{Aa}^{(+)} \Phi_{bx}^{a} \rangle$

  - post form $T_{fi} \approx \langle \chi_{Bb}^{(-)} \Phi_{Ax}^{B} | V_{Bb}^{(f)} - U_{Bb} | \chi_{Aa}^{(+)} \Phi_{bx}^{a} \rangle$

Spectroscopic Factors

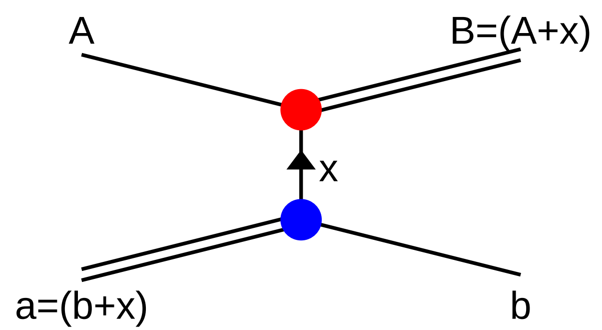
transfer reaction $A + a \to B + b$



- overlap functions $\hat{=}$ wave function of transferred particle

$$\Phi^{a}_{bx} = \langle \phi_b | \phi_a \rangle \qquad \Phi^{B}_{Ax} = \langle \phi_A | \phi_B \rangle$$

Spectroscopic Factors



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- approximation with spectroscopic amplitudes and single-particle wave functions

$$\Phi^a_{bx} \approx \mathcal{A}^a_{bx} \varphi^a_{bx}(\vec{r}_{bx}) \phi_x \quad \Phi^B_{Ax} \approx \mathcal{A}^B_{Ax} \varphi^B_{Ax}(\vec{r}_{Ax}) \phi_x \quad \langle \varphi^a_{bx} | \varphi^a_{bx} \rangle = \langle \varphi^A_{Bx} | \varphi^A_{Bx} \rangle = 1$$

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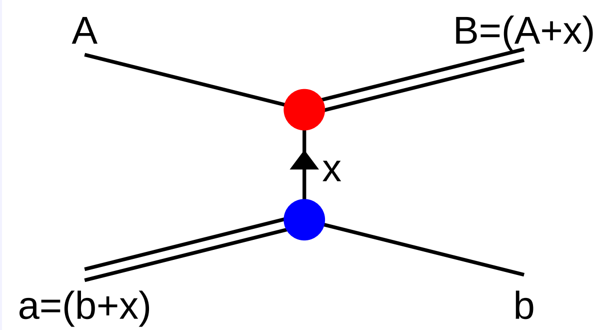

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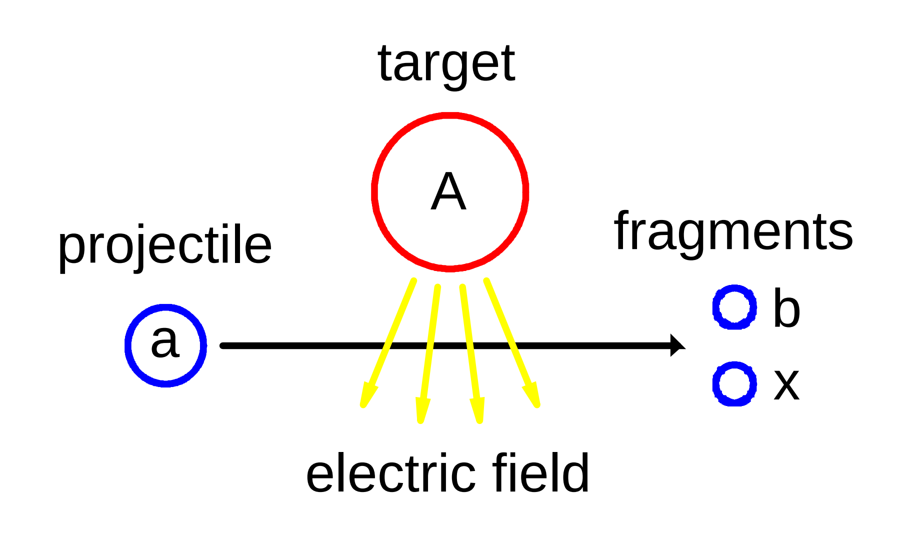
- T-matrix elements in DWBA
  - post form: $T_{(Bb)(Aa)} = \langle \Phi^B_{Ax} \chi^{(-)}_{Bb} | V_{Bb} - U_{Bb} | \Phi^a_{bx} \chi^{(+)}_{Aa} \rangle$
  - prior form: $T_{(Bb)(Aa)} = \langle \Phi^B_{Ax} \chi^{(-)}_{Bb} | V_{Aa} - U_{Aa} | \Phi^a_{bx} \chi^{(+)}_{Aa} \rangle$

- cross sections $d\sigma \propto \left| T_{(Bb)(Aa)} \right|^2 \Rightarrow d\sigma \approx \mathcal{S}^a_{bx} \mathcal{S}^B_{Ax} d\sigma_{\rm sp}$ factorization!

Coulomb Dissociation - Idea



radiative capture $b(x,\gamma)a$

detailed balance $\Updownarrow$

photo absorption $a(\gamma,x)b$

equivalent photons in Coulomb field of target A $\Updownarrow$

Coulomb dissociation $A(a,bx)A$

(G. Baur, H. Rebel, C. Bertulani, Nucl. Phys. A 458 (1986) 188)

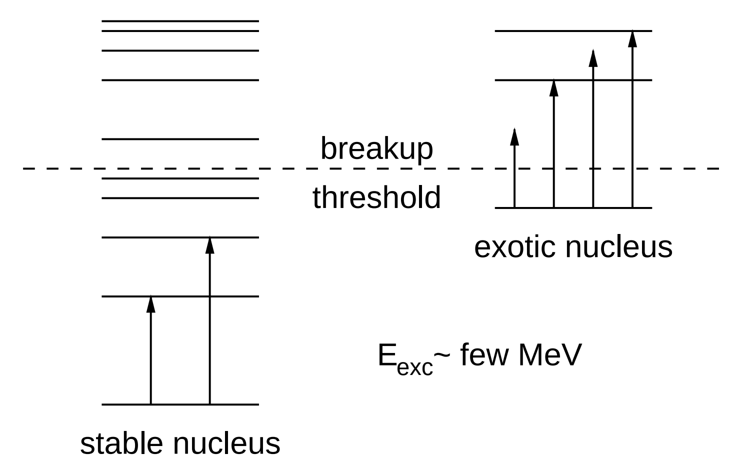
correspondence

(Fermi 1924, Weizsäcker-Williams 1932)

time-dependent electromagnetic field
of highly-charged nucleus A
during scattering of projectile a

$\Updownarrow$

spectrum of (virtual, equivalent) photons



only ground state transitions !

Coulomb Dissociation - Theory

Coulomb dissociation reaction: $a + A \rightarrow b + x + A$
with three-body final state in the continuum
$\Rightarrow$ only approximate theoretical treatment

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- **semiclassical methods**

– classical description of projectile-target relative motion
(valid for heavy targets if $\eta_{Aa} = Z_A Z_a e^2/(\hbar v) \gg 1$ with beam velocity v)

– time-dependent perturbation $V(t)$ of projectile system

– time-dependent perturbation theory

$\Rightarrow$ excitation amplitude a_{fi}

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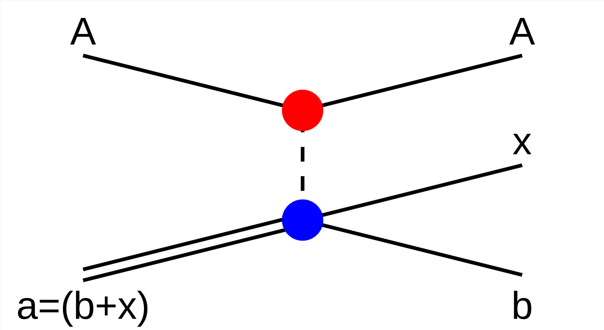
– time-dependent perturbation theory
$\Rightarrow$ excitation amplitude a_{fi}

- **quantal methods**

– valid for all projectile/target combinations and all beam energies
– time-independent scattering theory
$\Rightarrow$ T-matrix element T_{fi}

Coulomb Dissociation - Quantal Theory

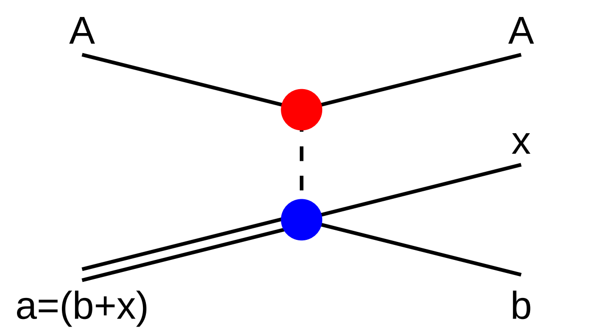


- prior-form distorted-wave Born approximation (DWBA)

$$T_{fi} = \langle \chi^{(-)}_{A(bx)} \phi_A \Psi^{(-)}_{bx} | V_{Aa} - U_{Aa} | \phi_A \phi_a \chi^{(+)}_{Aa} \rangle$$

- neglection of nuclear interaction in V_{Aa} and U_{Aa}

Coulomb Dissociation - Quantal Theory



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- neglection of nuclear interaction in V_{Aa} and U_{Aa}

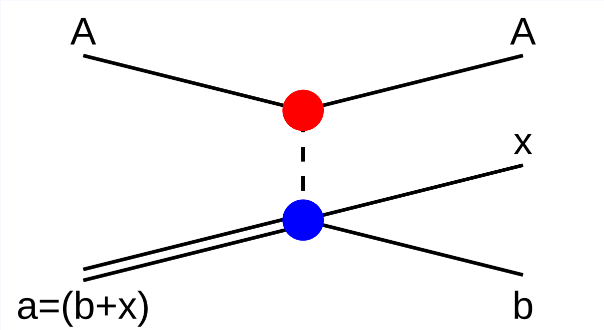
- multipole expansion of Coulomb potential in far-field approximation ($r_{bx} < r_{Aa}$)

$$V_{Aa} - U_{Aa} = \frac{Z_A Z_b e^2}{|\vec{r}_b - \vec{r}_A|} + \frac{Z_A Z_x e^2}{|\vec{r}_x - \vec{r}_A|} - \frac{Z_A Z_a e^2}{|\vec{r}_a - \vec{r}_A|} \approx 4\pi Z_A e \sum_{\lambda\mu} \frac{Z^{(\lambda)}_{\mathrm{eff}} e}{2\lambda+1} \frac{r^{\lambda}_{bx}}{r^{\lambda+1}_{Aa}} Y_{\lambda\mu}(\hat{r}_{bx}) Y^*_{\lambda\mu}(\hat{r}_{Aa})$$

with effective charge numbers $Z^{(\lambda)}_{\mathrm{eff}} = Z_b \left(\frac{m_x}{m_b+m_x}\right)^{\lambda} + Z_x \left(-\frac{m_b}{m_b+m_x}\right)^{\lambda}$

and relative coordinates $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, $\vec{r}_{Aa} = \vec{r}_A - \vec{r}_a$

Coulomb Dissociation - Quantal Theory



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and relative coordinates $\vec{r}_{bx} = \vec{r}_b - \vec{r}_x$, $\vec{r}_{Aa} = \vec{r}_A - \vec{r}_a$

$\Rightarrow$ factorization of T-matrix element

$$T_{fi} \approx Z_A e \sum_{\lambda\mu} \frac{4\pi}{2\lambda+1} \langle \Psi^{(-)}_{bx} | \underbrace{Z^{(\lambda)}_{\mathrm{eff}} e r^{\lambda}_{bx} Y_{\lambda\mu}(\hat{r}_{bx})}_{\mathcal{M}(E\lambda\mu)} | \phi_a \rangle \langle \chi^{(-)}_{A(bx)} | r^{-\lambda-1}_{Aa} Y^{*}_{\lambda\mu}(\hat{r}_{Aa}) | \chi^{(+)}_{Aa} \rangle$$

$\mathcal{M}(E\lambda\mu)$ electric multipole transition operator

$\Rightarrow$ transition matrix element $\times$ quantal Coulomb integral

Coulomb Dissociation - Cross Section

Coulomb dissociation cross section
(with angular integration over relative momentum between fragments)

$$\Rightarrow \quad \frac{d^2\sigma}{dE_{bx}d\Omega_{aA}} = \frac{1}{E_\gamma}\sum_{\pi\lambda} \sigma_{\pi\lambda}(a+\gamma \to b+x)\,\frac{dn_{\pi\lambda}}{d\Omega_{aA}} \qquad \pi = E, M \qquad \lambda = 1, 2, \ldots$$

- photo absorption cross section $\sigma_{\pi\lambda}(a+\gamma \to b+x)$
- virtual photon numbers $\frac{dn_{\pi\lambda}}{d\Omega_{aA}}$

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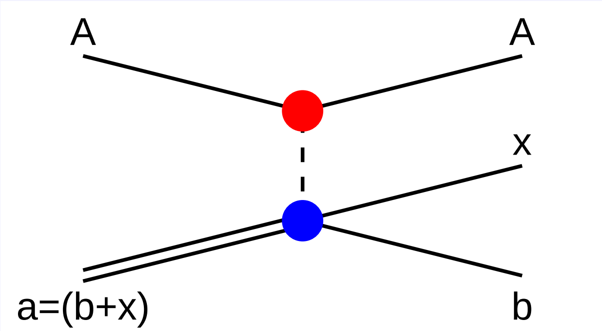
- photo absorption cross section $\sigma_{\pi\lambda}(a+\gamma \to b+x)$
- virtual photon numbers $\frac{dn_{\pi\lambda}}{d\Omega_{aA}}$ that depend on kinematics:
  - ◦ scattering angle ϑ_{aA}/impact parameter b
  - ◦ projectile velocity v
  - ◦ excitation energy $E_\gamma = \hbar\omega$

  calculation in
  - ◦ non-relativistic approximation with Coulomb (hyperbolic) scattering trajectories
  - ◦ relativistic approximation with straight-line trajectories

  $\Rightarrow$ E2 enhancement $\frac{dn_{E2}}{d\Omega_{aA}} \Big/ \frac{dn_{E1}}{d\Omega_{aA}} \approx \frac{4\hbar^2 c^2}{E_\gamma^2 b^2}$ M1 suppression $\frac{dn_{M1}}{d\Omega_{aA}} \Big/ \frac{dn_{E1}}{d\Omega_{aA}} \approx \frac{v^2}{c^2}$

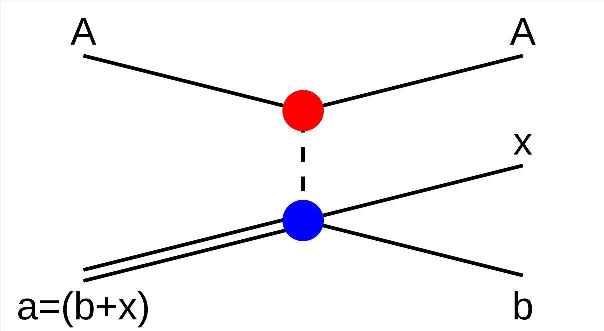
Coulomb Dissociation - Relation of Cross Sections



- Coulomb dissociation cross section

$$\frac{d^2\sigma}{dE_{bx}d\Omega_{Aa}} = \frac{1}{E_\gamma}\sum_{\pi\lambda}\sigma_{\pi\lambda}(a+\gamma\to b+x)\frac{dn_{\pi\lambda}}{d\Omega_{Aa}}$$

Coulomb Dissociation - Relation of Cross Sections



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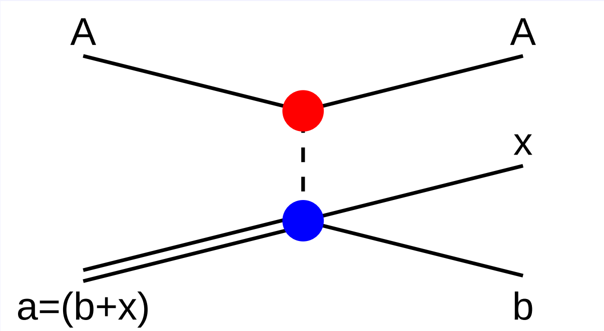
- theorem of detailed balance

$$\sigma_{\pi\lambda}(a+\gamma\to b+x) = \frac{(2J_b+1)(2J_x+1)}{2(2J_a+1)}\frac{k_{bx}^2}{k_\gamma^2}\sigma_{\pi\lambda}(b+x\to a+\gamma)$$

with photo absorption and radiative capture cross sections

Coulomb Dissociation - Relation of Cross Sections



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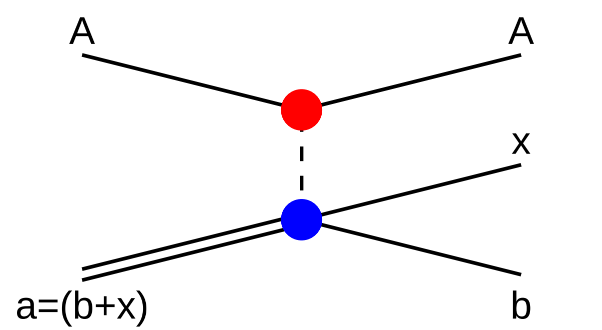
with photo absorption and radiative capture cross sections

- phase space factor $\dfrac{k_{bx}^2}{k_\gamma^2} = \dfrac{2\mu_{bx}c^2E_{bx}}{(E_{bx}+S_{bx})^2} \gg 1$ for not too small E_{bx}

- virtual photon numbers $\dfrac{dn_{\pi\lambda}}{d\Omega_{Aa}} \gg 1$ for large Z_A and for not too high E_{bx} $(\hat{=}\,\xi)$

$\Rightarrow$ **large Coulomb dissociation cross sections**



- **adiabaticity parameter**

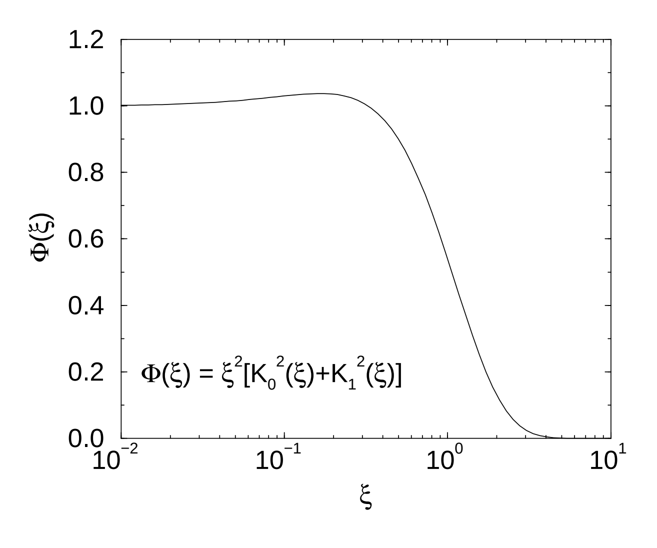
$$\xi = \frac{\omega b}{\gamma v} = \frac{\text{duration of scattering process}}{\text{excitation period}}$$

$\xi = 0$: sudden excitation

$\xi \gg 1$: adiabatic excitation

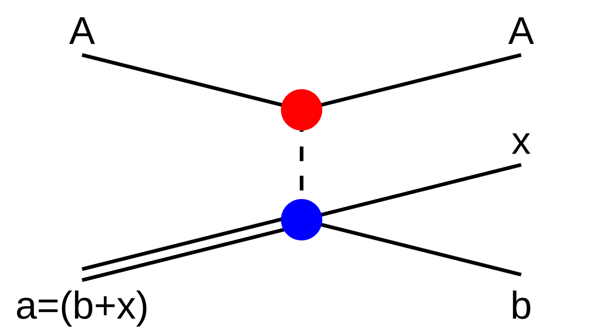
$\xi \approx 1 \Rightarrow E_{\text{exc}}^{\text{max}} \approx \gamma v \hbar / b$

virtual photon spectrum (E1)



(Fermi 1924, Weizsäcker-Williams 1932)

Coulomb Dissociation - Characteristic Parameters



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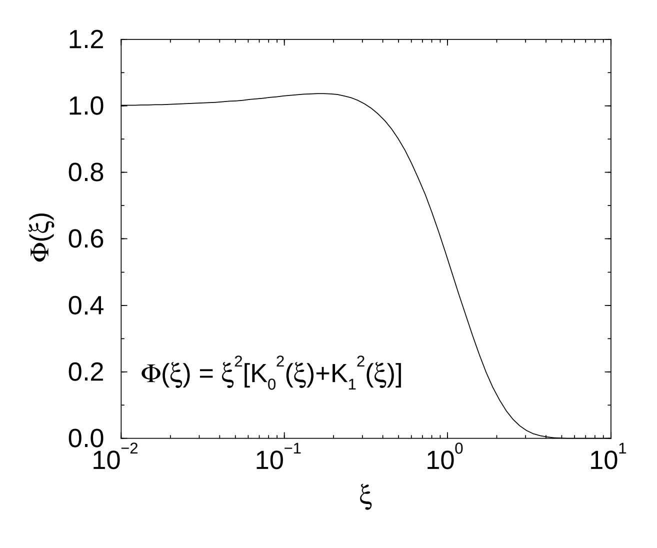

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$\xi = 0$: sudden excitation

$\xi \gg 1$: adiabatic excitation

$\xi \approx 1 \Rightarrow E_{\text{exc}}^{\text{max}} \approx \gamma v \hbar / b$

- **strength parameter**

$$\chi = \frac{Z_A e \langle f || \mathcal{M}(\pi\lambda) || i \rangle}{\hbar v b^{\lambda}}$$

$Z_A e$ target charge

$\mathcal{M}(\pi\lambda)$ multipole operator

χ small $\Rightarrow$ first-order perturbation theory sufficient

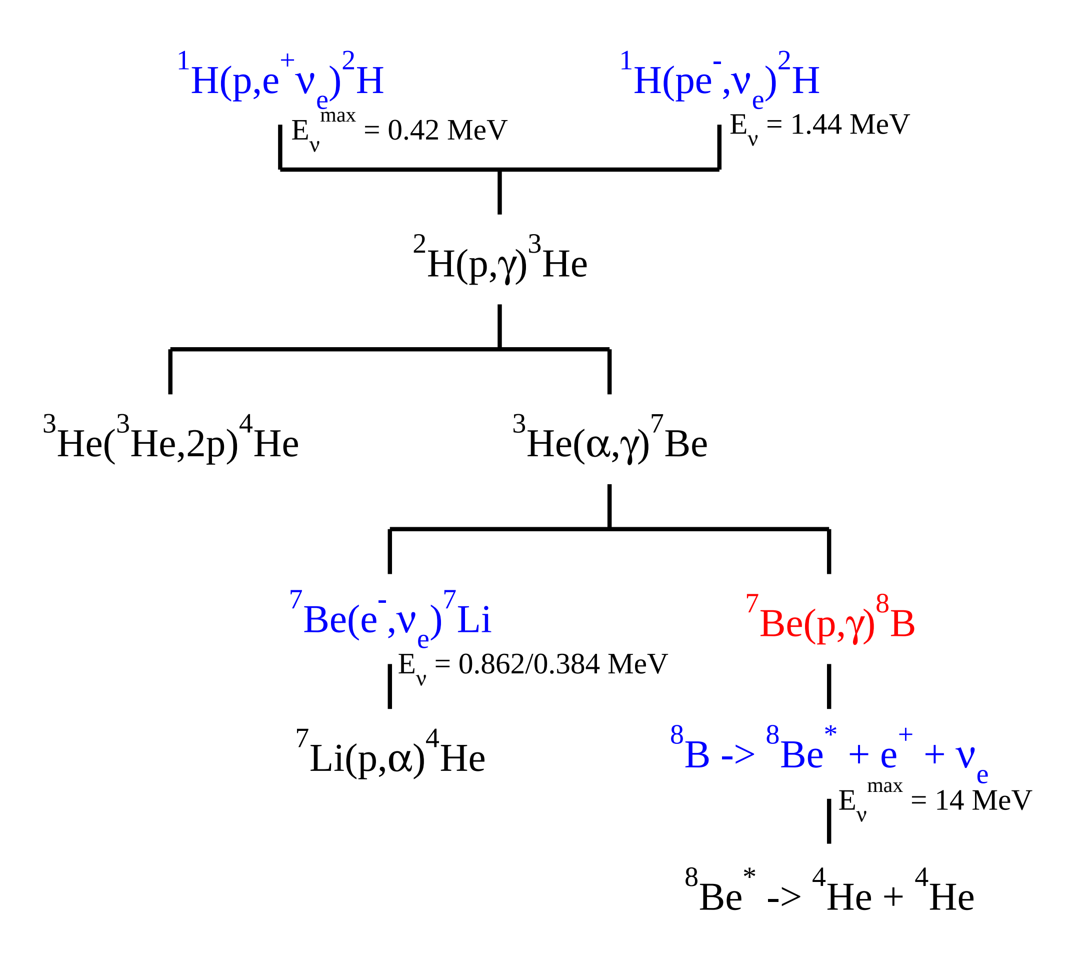
χ large $\Rightarrow$ higher-order effects

Coulomb Dissociation - Example: ^{8}B

^{8}B and solar neutrinos

- pp chains: main source of solar energy production
- flux of high-energy neutrinos proportional to synthesized ^{8}B
- precise knowledge of ^{7}Be(p,γ)^{8}B S factor $S_{17}(E)$ in Gamov window ($E \approx 20$ keV) required
- solar neutrino problem solved with neutrino oscillation
- more precise direct capture data available recently

$\Rightarrow$ **test case** for Coulomb dissociation method see lecture by K. Sümmerer

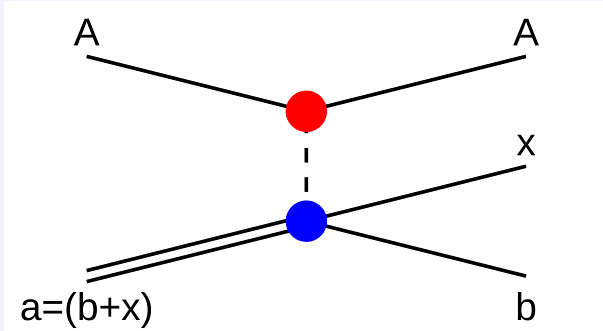


Coulomb Dissociation - Higher-Order Effects I

interaction of fragments in final state
with target Coulomb field after breakup

$\Rightarrow$ "post-acceleration" $\hat{=}$ multi-photon exchange

$\Rightarrow$ Coulomb dissociation cross section
not proportional to photo absorption cross section



Stefan Typel

Coulomb Dissociation - Higher-Order Effects I

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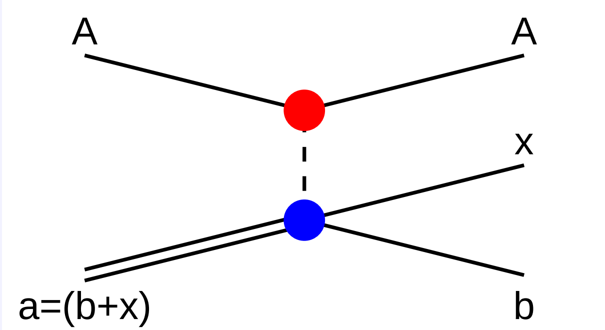

theoretical approaches

- semiclassical description
  - higher-order perturbation theory
  - full dynamical calculation (solving the time-dependent Schrödinger equation)
  - sudden approximation

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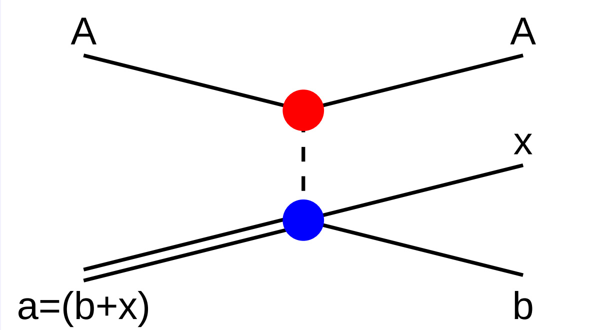

theoretical approaches

- semiclassical description
  - higher-order perturbation theory
  - full dynamical calculation (solving the time-dependent Schrödinger equation)
  - sudden approximation
- quantal description
  - post-form DWBA $T_{fi} = \langle \chi_{Ab}^{(-)} \chi_{Ax}^{(-)} \phi_A \phi_b \phi_x e^{i\vec{k}_{bx}\cdot\vec{r}_{bx}} | V_{bx} | \phi_A \phi_a \chi_{Aa}^{(+)} \rangle$

    first order in V_{bx}, all orders in V_{Ab} and V_{Ax}

    $\Rightarrow$ factorization, Bremsstrahlung integrals

Coulomb Dissociation - Higher-Order Effects II

Coulomb excitation of ^{11}Be
from ground state ($\frac{1}{2}^{+}$, s-wave, $E_0 = -504$ keV)
to first excited state ($\frac{1}{2}^{-}$, p-wave, $E_1 = -184$ keV)

- excitation amplitude in sudden approximation

  $a_{\text{sudden}} = \langle f | \exp(-i\vec{q}_{\text{Coul}} \cdot \vec{r}) | i \rangle$ with

  momentum transfer $\hbar\vec{q}_{\text{Coul}} = \frac{2ZZ_{\text{eff}}^{(1)}e^2}{vb}\vec{e}_z$

Coulomb Dissociation - Higher-Order Effects II

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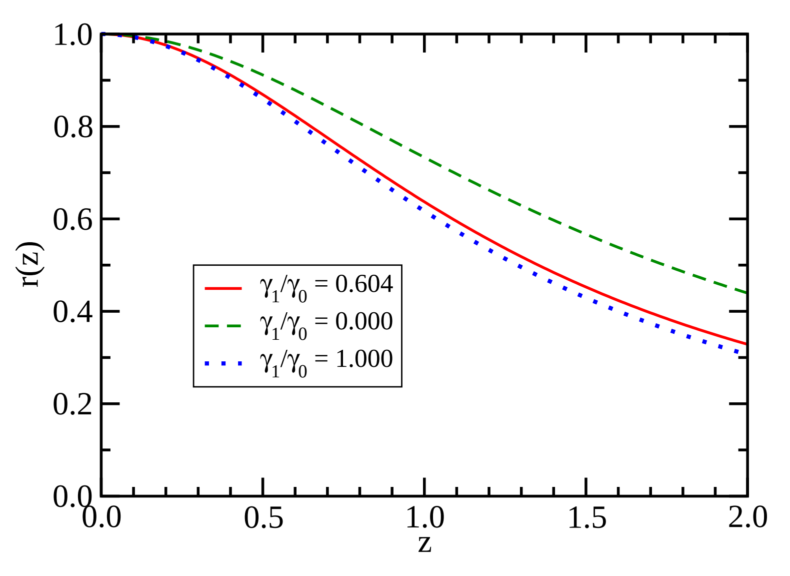
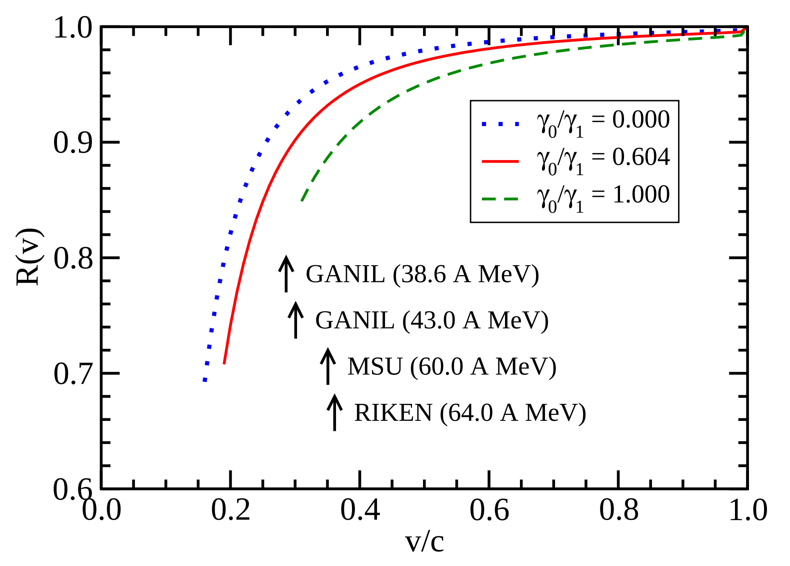
- reduction factors
  - probability $r(z) = \frac{|a_{\text{sudden}}|^2}{|a_1|^2}$ depends on

    $z = \frac{q_{\text{Coul}} R}{\gamma_0 + \gamma_1}$ and $\gamma_1/\gamma_0 = \sqrt{E_1/E_0}$ (analytical)
  - cross section $R(v) = \sigma^{(\infty)}/\sigma^{(1)}$

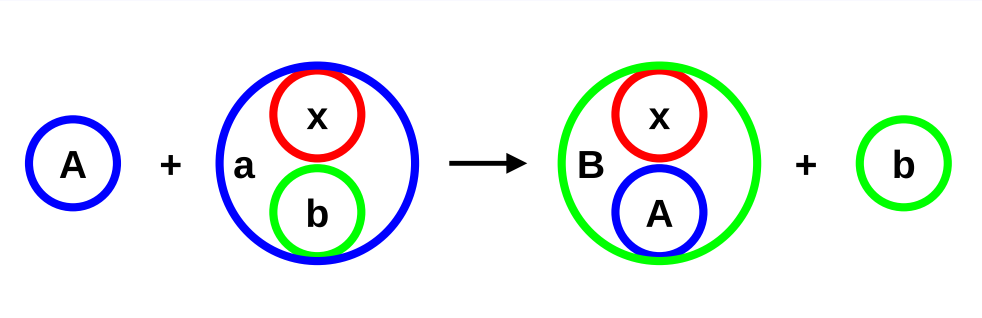
  $\Rightarrow$ simple scaling laws

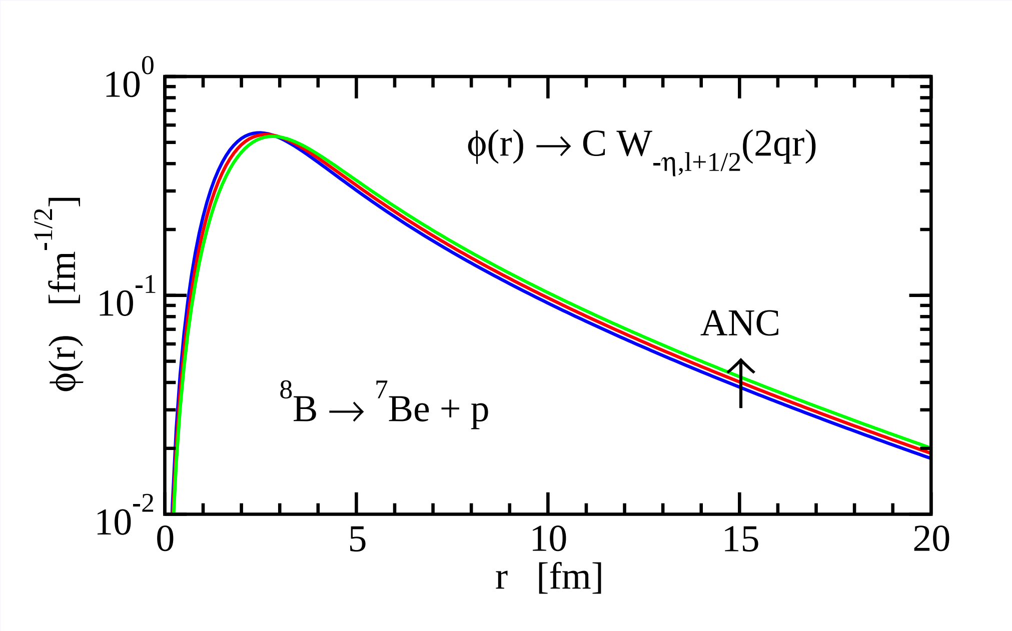
(S. Typel and G. Baur, Eur. Phys. J. A 38 (2008) 355)





ANC Method - Idea





extract asymptotic normalization coefficient (**ANC**)

for breakup of nucleus B into $A + x$
or nucleus a into $b + x$

from cross section of transfer reaction

$$A + a \rightarrow B + b$$

with $a = b + x$ and $B = A + x$

$$\Downarrow$$

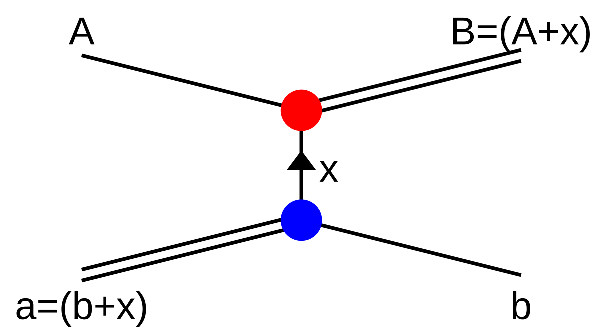
calculate astrophysical S factor $S(E)$

in the limit $E \rightarrow 0$

(H.M. Xu et al., Phys. Rev. Lett. 73 (1994) 2027)

ANC Method - Theory I

- T-matrix element in post-form DWBA
- replace exact overlap functions by asymptotic form with asymptotic normalization coefficients (ANCs) and Whittaker functions



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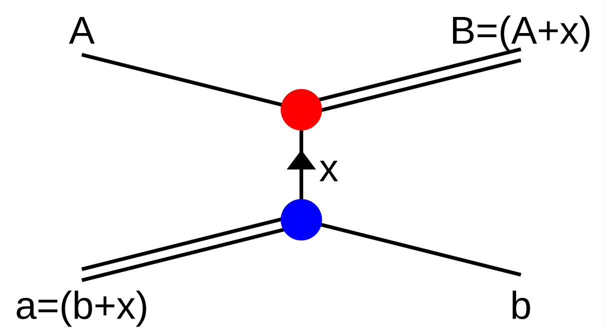

- **overlap functions** ($\hat{=}$ wave function of transferred particle, neglecting spins)

$$\langle \phi_b | \phi_a \rangle \approx \frac{C_{bx}^{a}(l_a)}{r_{bx}} W_{-\eta_{bx}, l_a+1/2}(2q_{bx} r_{bx}) Y_{l_a m_a}(\hat{r}_{bx}) \phi_x$$

$$\langle \phi_A | \phi_B \rangle \approx \frac{C_{Ax}^{B}(l_B)}{r_{Ax}} W_{-\eta_{Ax}, l_B+1/2}(2q_{Ax} r_{Ax}) Y_{l_B m_B}(\hat{r}_{Ax}) \phi_x$$

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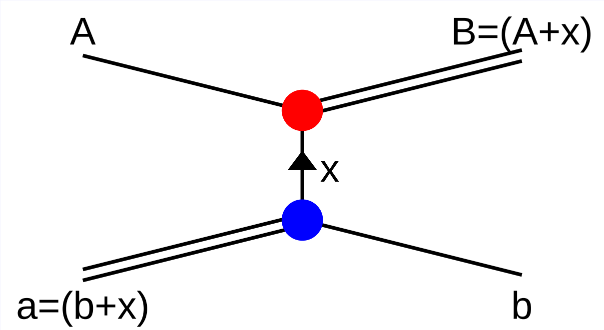

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$$\langle \phi_A | \phi_B \rangle \approx \frac{C^B_{Ax}(l_B)}{r_{Ax}} W_{-\eta_{Ax}, l_B+1/2}(2q_{Ax}r_{Ax}) Y_{l_B m_B}(\hat{r}_{Ax}) \phi_x$$

- **cross section** of transfer reaction to bound state

$$\frac{d\sigma}{d\Omega_{Bb}} = \left|C^a_{bx}\right|^2 \left|C^B_{Ax}\right|^2 \frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$ with reduced DWBA cross section $$\frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$

ANC Method - Theory I

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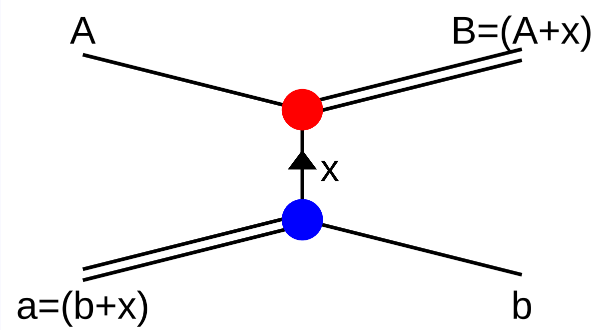
- replace exact overlap functions by asymptotic form with asymptotic normalization coefficients (ANCs) and Whittaker functions



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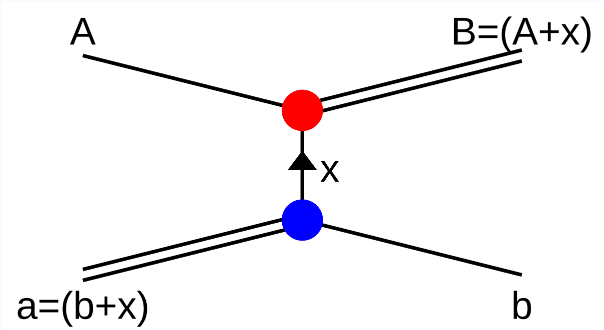
- **cross section** of transfer reaction to bound state

$$\frac{d\sigma}{d\Omega_{Bb}} = \left| C^a_{bx} \right|^2 \left| C^B_{Ax} \right|^2 \frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$ with reduced DWBA cross section $$\frac{d\tilde{\sigma}}{d\Omega_{Bb}}$$

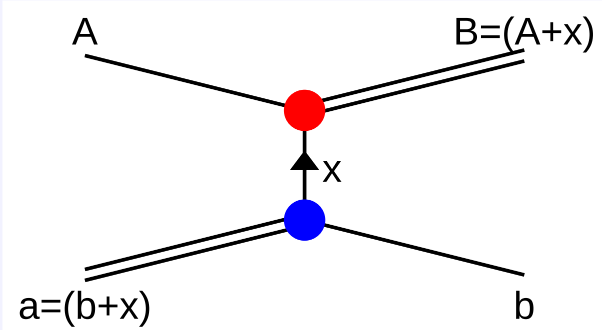
- two ANCs appear corresponding to two poles in diagram
- ANC/Whittaker functions replace spectroscopic factors/full single-particle wave functions in conventional DWBA

ANC Method - Theory II

- approximations only valid for weakly bound states/ peripheral reactions
- precise optical potentials for $A + a$ and $B + b$ scattering required
- one additional ANC needed



ANC Method - Theory II



- approximations only valid for weakly bound states/ peripheral reactions
- precise optical potentials for $A+a$ and $B+b$ scattering required
- one additional ANC needed

⇒ calculate low-energy S factor of capture reaction $b(x,\gamma)a$ numerically

with extracted ANC $C_{bx}^{a}(l_a)$ and asymptotic wave function

- $|C_{bx}^{a}|^2 \Leftrightarrow S(0)$ unique relation?
- effect of final-state interaction V_{bx}?

  ⇒ systematic model calculations

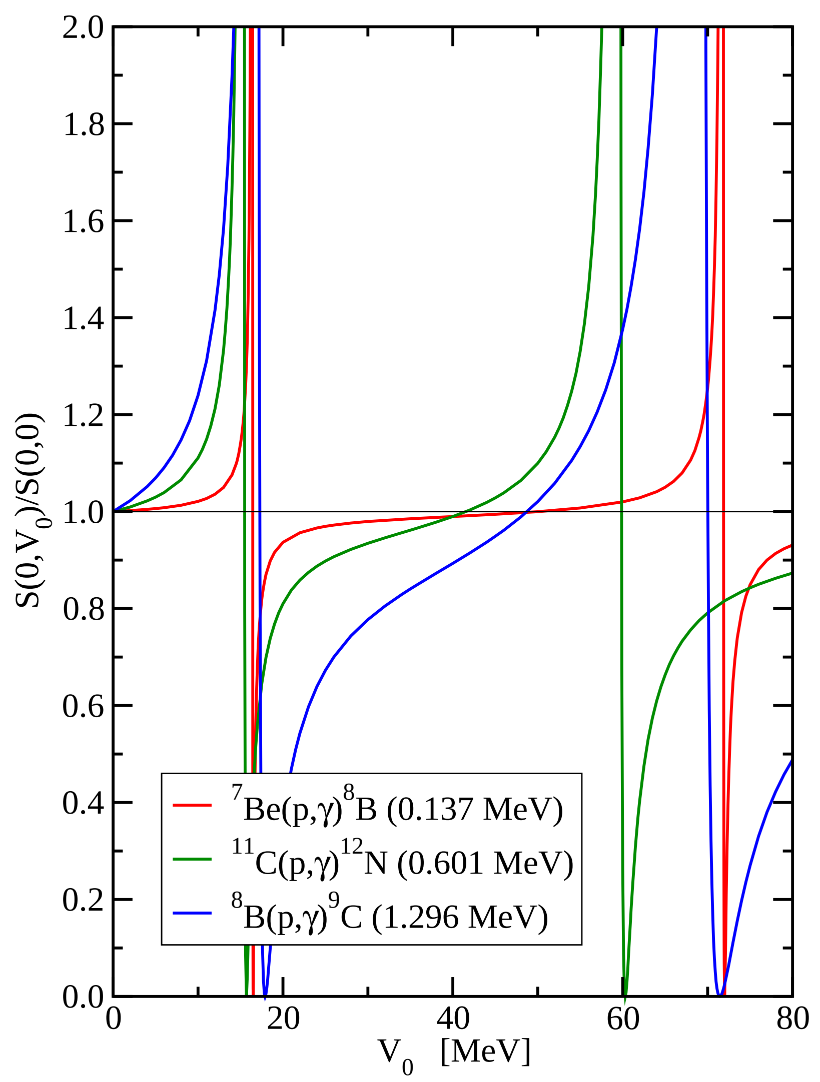
ANC Method - Continuum Interaction

- effects of interaction in continuum states
  - modification of shape of cross section, S factor (i.e. energy dependence)
  - change of $S(0)$ even though $\delta \to 0$
- calculation of zero-energy S factor $S(0)$ in single-particle model with Woods-Saxon potential with different depths V_0

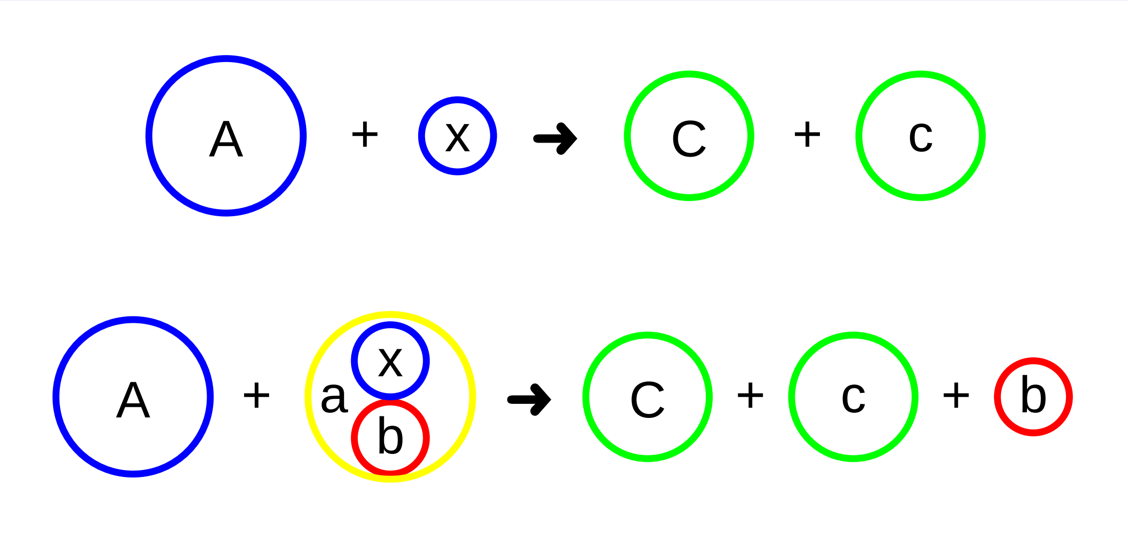
ANC Method - Continuum Interaction

- effects of interaction in continuum states
  - modification of shape of cross section, S factor (i.e. energy dependence)
  - change of $S(0)$ even though $\delta \to 0$
- calculation of zero-energy S factor $S(0)$ in single-particle model with Woods-Saxon potential with different depths V_0
- example: $E1$ $s \to p$ wave capture for different nuclei with proton+core structure
  $\Rightarrow$ stronger variation of $S(0)$ with V_0 with larger proton separation energy
- simple relation ANC $\Leftrightarrow$ $S(0)$ only correct for weakly bound nuclei

S. Typel and G. Baur, Nucl. Phys. A 759 (2005) 245



Trojan-Horse Method - Idea



replace **two-body reaction**

$$A + x \rightarrow C + c$$

by **three-body reaction**

$$A + a \rightarrow C + c + b$$

with **Trojan horse** $a = b + x$

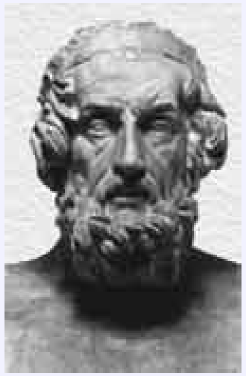
and **spectator** b

- small momentum transfer to spectator
  $\Rightarrow$ quasi-free scattering dominates
- large relative energy of system $A + a$
  $\Rightarrow$ no suppression of cross section
  $\Rightarrow$ no electron screening
- small relative energies of system $A + x$ accessible
  $\Rightarrow$ application to nuclear astrophysics

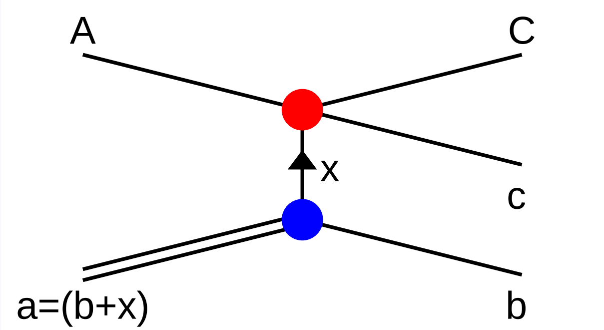
(G. Baur, Phys. Lett. B 178 (1986) 35)

$\ldots\ \kappa\epsilon\kappa\alpha\lambda\upsilon\mu\mu\epsilon\nu o\iota\ \iota\pi\pi o.$

Homer, Odyssey VIII, 503



Trojan-Horse Method - Theory I

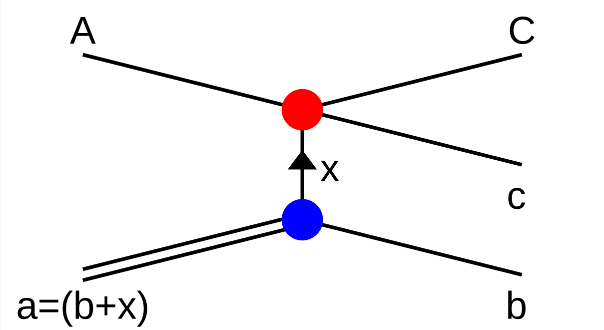


- T-matrix element in post-form DWBA with $B = C + c$

$$T_{(Bb)(Aa)} = \langle \phi_B \phi_b \chi_{Bb}^{(-)} | V_{Bb} - U_{Bb} | \phi_a \phi_A \chi_{Aa}^{(+)} \rangle$$

- use asymptotic form of scattering wave function $\phi_B = \Psi_{Cc}^{(-)}$ in reaction channel $C + c \to A + x$ ("surface approximation")

Trojan-Horse Method - Theory I



- T-matrix element post-form DWBA with $B = C + c$

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- use asymptotic form of scattering wave function $\phi_B = \Psi^{(-)}_{Cc}$ in reaction channel $C + c \to A + x$ ("surface approximation")

$\Rightarrow$ **overlap function** ($\hat{=}$ wave function of transferred particle, neglecting spins)

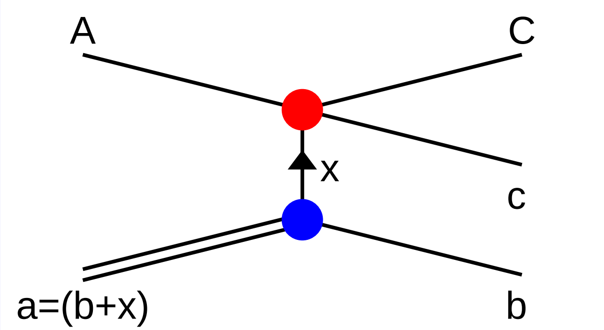
$$\langle \phi_A | \Psi^{(-)}_{Cc} \rangle \approx \frac{4\pi}{k_{Cc} r_{Ax}} \sqrt{\frac{v_{Cc}}{v_{Ax}}} \sum_{lm} \xi^*_l(r_{Ax}) i^l Y_{lm}(\hat{r}_{Ax}) Y^*_{lm}(\hat{k}_{Cc}) \phi_x$$

with $\xi_l(r_{Ax}) = \frac{1}{2i} \left[S^l_{AxCc} u^{(+)}_l(\eta_{Ax}; k_{Ax} r_{Ax}) - \delta_{AxCc} u^{(-)}_l(\eta_{Ax}; k_{Ax} r_{Ax}) \right]$

and S-matrix element S^l_{AxCc} of reaction $C(c,x)A$

(S. Typel and G. Baur, Ann. Phys. (N.Y.) 305 (2003) 228)

Trojan-Horse Method - Theory II

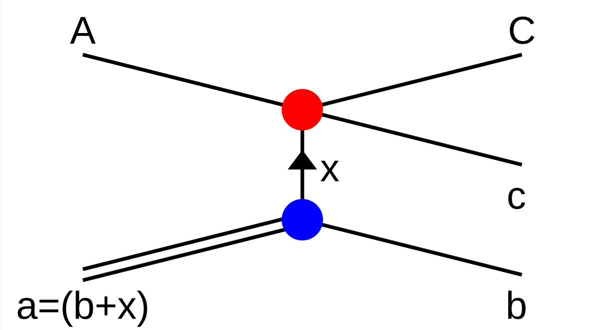


- post-form DWBA T-matrix element with surface approximation for $ax \neq Cc$
  $\Rightarrow$ factorization

$$T_{(Bb)(Aa)} \propto \sum_{lm} S^{l*}_{AxCc}$$

$$\times \langle \frac{u_l^{(+)}(\eta_{Ax};k_{Ax}r_{Ax})}{k_{Cc}r_{Ax}} Y_{lm}(\hat{r}_{Ax}) \phi_x \phi_b \chi^{(-)}_{Bb} | V_{Bb} - U_{Bb} | \phi_a \chi^{(+)}_{Aa} \rangle$$

Trojan-Horse Method - Theory II



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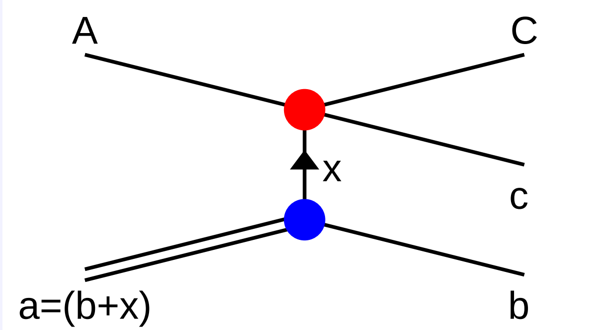
  $\Rightarrow$ **cross section** of transfer reaction to continuum (single channel, $Ax \neq Cc$)

$$\frac{d^3\sigma}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}} = \left| S^l_{AxCc} \right|^2 \frac{d^3\tilde{\sigma}_l}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}}$$

with reduced DWBA cross section

Trojan-Horse Method - Theory II



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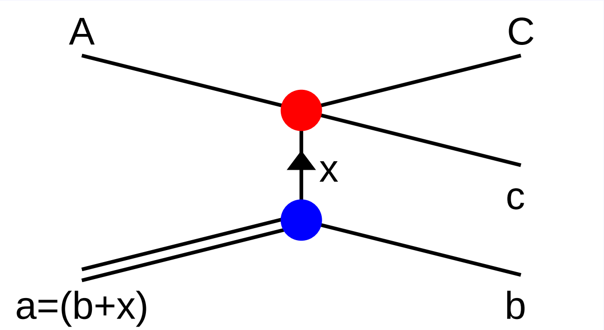
with reduced DWBA cross section

- S-matrix elements S^l_{AxCc} determine cross section

$$\frac{d\sigma}{d\Omega_{Ax}}(C + c \rightarrow A + x) = \frac{\pi}{k^2_{bx}} \left| \sum_l S^l_{AxCc} Y_{l0}(\hat{r}_{Ax}) \right|^2$$

Trojan-Horse Method - Theory III

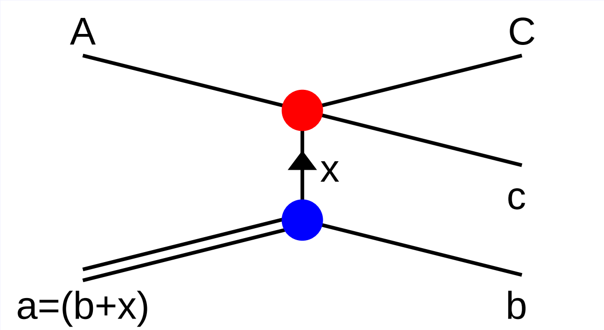


additional approximations

(not necessary in general, but convenient)

- potential $V_{Bb} - U_{Bb} = V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi_{Bb}^{(-)}$, $\chi_{Aa}^{(+)}$

Trojan-Horse Method - Theory III



additional approximations
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- potential $V_{Bb} - U_{Bb} = V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi^{(-)}_{Bb}$, $\chi^{(+)}_{Aa}$
- T-matrix element in PWBA with surface approximation

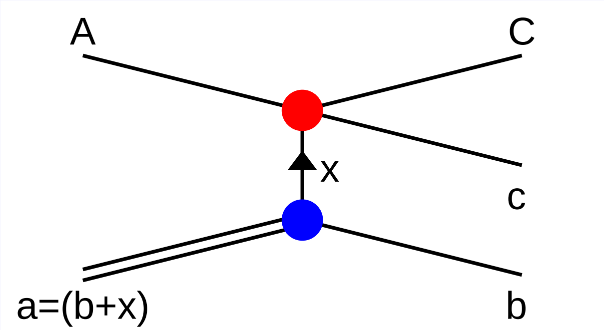
$$T_{(Bb)(Aa)} \propto \sum_{lm} S^{l*}_{AxCc} \langle \phi_x \phi_b \exp\left(i\vec{Q}_{Bb}\cdot\vec{r}_{bx}\right)|V_{xb}|\phi_a\rangle$$
$$\times \langle \frac{u_l^{(+)}(\eta_{Ax};k_{Ax}r_{Ax})}{k_{Cc}r_{Ax}} Y_{lm}(\hat{r}_{Ax})|\exp\left(i\vec{Q}_{Aa}\cdot\vec{r}_{Ax}\right)\rangle$$

with momentum transfers

$$\vec{Q}_{Aa} = \vec{k}_{Aa} - \frac{\mu_{Ax}}{m_x}\vec{k}_{Bb} \qquad \vec{Q}_{Bb} = \vec{k}_{Bb} - \frac{\mu_{bx}}{m_x}\vec{k}_{Aa}$$

- factorization with three factors:
  - two factors for two poles in diagram
  - additional matrix element, depends on kinematics (η_{Ax}!)

Trojan-Horse Method - Theory IV



additional approximations
(not necessary in general, but convenient)

- potential $V_{Bb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi^{(-)}_{Bb}$, $\chi^{(+)}_{Aa}$

$\Rightarrow$ **cross section** of transfer reaction to continuum (single channel)

$$\frac{d^3\sigma}{d\Omega_{Bb} d\Omega_{Cc} dE_{Cc}} = K\, W(\vec{Q}_{Bb}) \frac{d\sigma_l}{d\Omega}(Ax \to Cc)\, T_l(k_{Ax})$$

with kinematic factor K

- **momentum distribution** $W(\vec{Q}_{Bb}) = |\tilde{\Phi}^a_{bx}(\vec{Q}_{Bb})|^2$

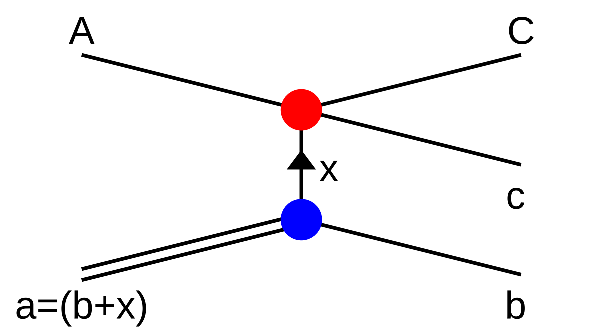
  depending on momentum transfer to spectator $b \Rightarrow$ quasi-free scattering conditions

- **cross section** $\frac{d\sigma_l}{d\Omega}(Ax \to Cc)$ of two-body reaction
- **penetration factor** $T_l(k_{Ax}) \approx k^3_{Ax} \exp(2\pi\eta_{Ax})$

Trojan-Horse Method - Theory IV

additional approximations
(not necessary in general, but convenient)

- potential $V_{Bb} - U_{Bb} \approx V_{xb}$
- plane waves for distorted waves $\chi_{Bb}^{(-)}$, $\chi_{Aa}^{(+)}$



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  depending on momentum transfer to spectator $b \Rightarrow$ quasi-free scattering conditions

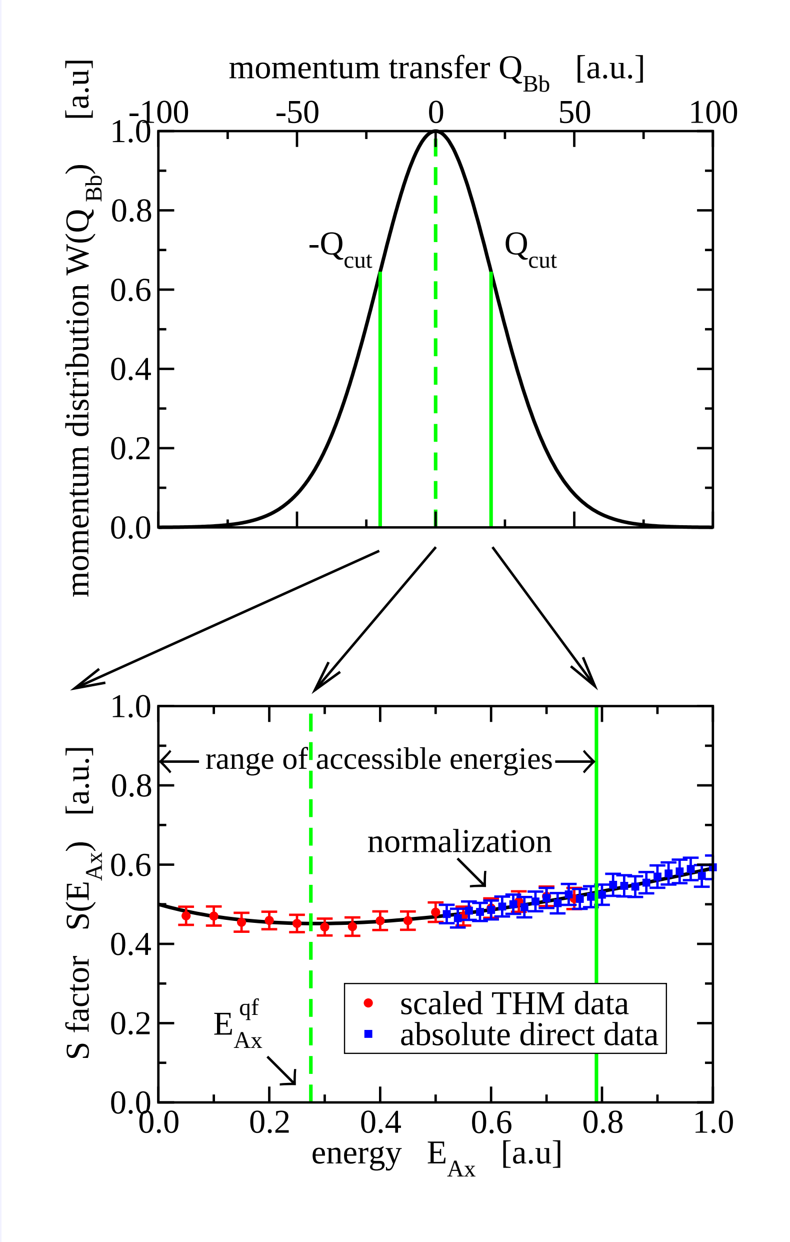
- **cross section** $\frac{d\sigma_l}{d\Omega}(Ax \to Cc)$ of two-body reaction

- **penetration factor** $T_l(k_{Ax}) \approx k_{Ax}^3 \exp(2\pi\eta_{Ax})$

$\Rightarrow$ cancels suppression of two-body cross section by Coulomb barrier for $E_{Ax} \to 0$

Trojan-Horse Method - Application

- selection of Trojan horse $a = b + x$
  (e.g. ^{2}H $= n + p$, ^{6}Li $= \alpha + d$, $\ldots$)
  with binding energy $\epsilon_a > 0$ and
  well known ground state wave function
  $\Rightarrow$ momentum distribution $W(\vec{Q}_{Bb})$
- width of momentum distribution W
  $\Leftrightarrow$ Fermi motion of x inside a

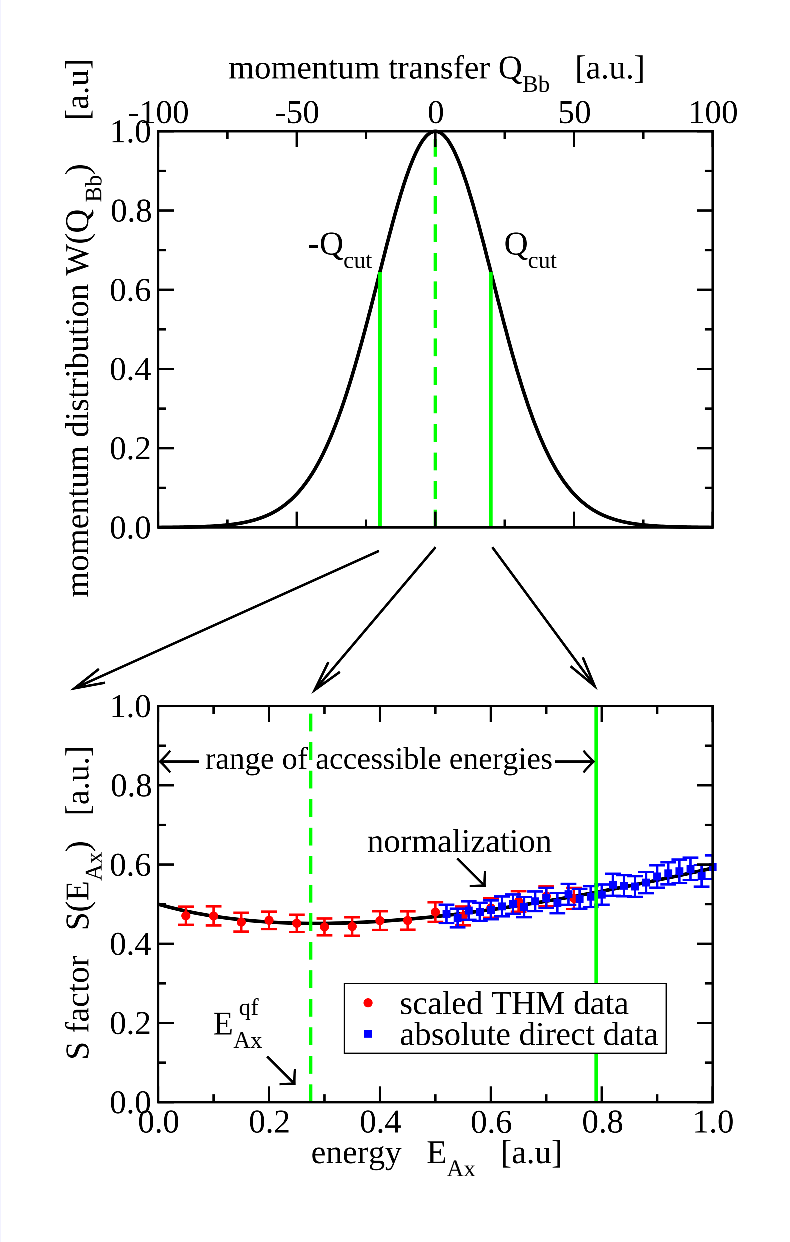


Trojan-Horse Method - Application

- selection of Trojan horse $a = b + x$
  (e.g. ^{2}H $= n + p$, ^{6}Li $= \alpha + d$, $\ldots$)
  with binding energy $\epsilon_a > 0$ and
  well known ground state wave function
  $\Rightarrow$ momentum distribution $W(\vec{Q}_{Bb})$
- width of momentum distribution W
  $\Leftrightarrow$ Fermi motion of x inside a
- condition $\vec{Q}_{Bb} = 0$ defines
  "quasi-free energy" in $A + x$ system

$$E_{Ax}^{qf} = E_{Aa}\left(1 - \frac{\mu_{Aa}}{\mu_{Bb}}\frac{\mu_{bx}^2}{m_x^2}\right) - \epsilon_a \ll E_{Aa}$$

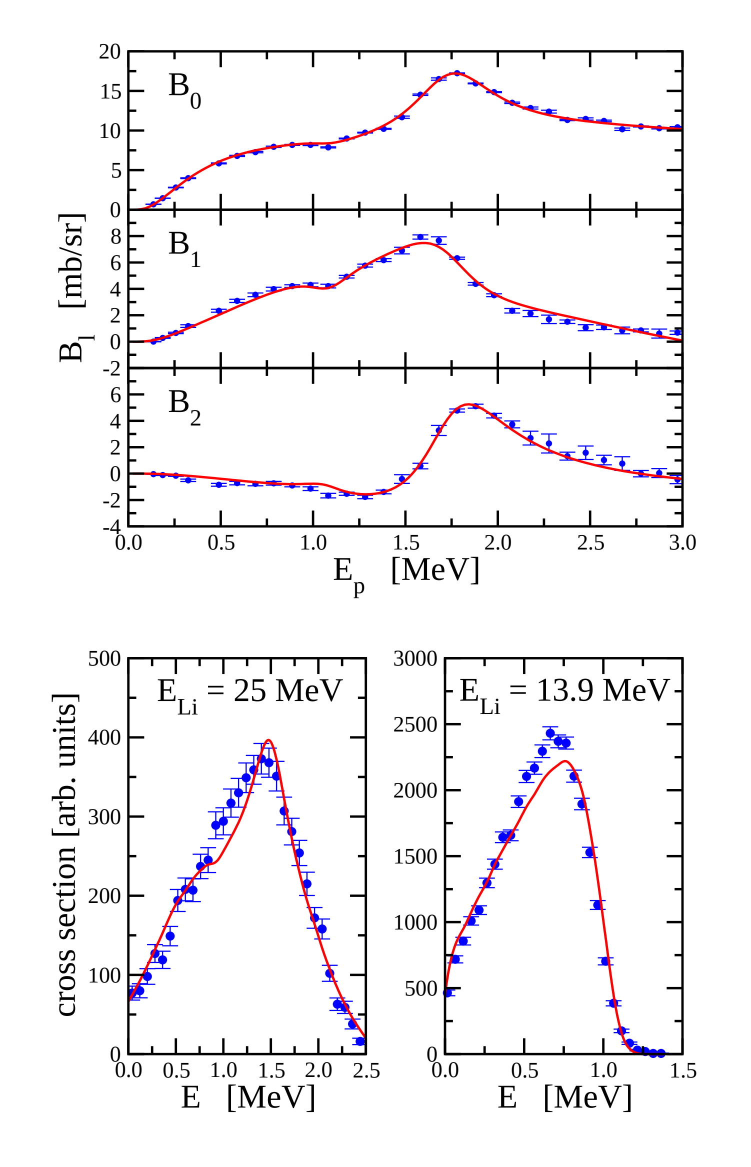
- cutoff in $\vec{Q}_{Bb}$ determines range of
  accessible energies E_{Ax} around E_{Ax}^{qf}
- small momentum transfer
  $\Rightarrow$ dominance of quasi-free process
- normalization of cross section to direct data
  at higher E_{Ax}



Trojan-Horse Method - Example: ^{6}Li(p,α)^{3}He

- **direct reaction**: ^{6}Li(p,α)^{3}He
  - experimental data
    (J. Elwyn et al., Phys. Rev. C 20 (1979) 1084)
  - differential cross section

    $$d\sigma/d\Omega = \sum_l B_l P_l(\cos\theta)$$
  - non-resonant s wave and resonant p wave contribution
  - S matrix from R-matrix fit
    $\Rightarrow$ simulation of THM experiment



 Stefan Typel

Trojan-Horse Method - Example: ^{6}Li(p,α)^{3}He

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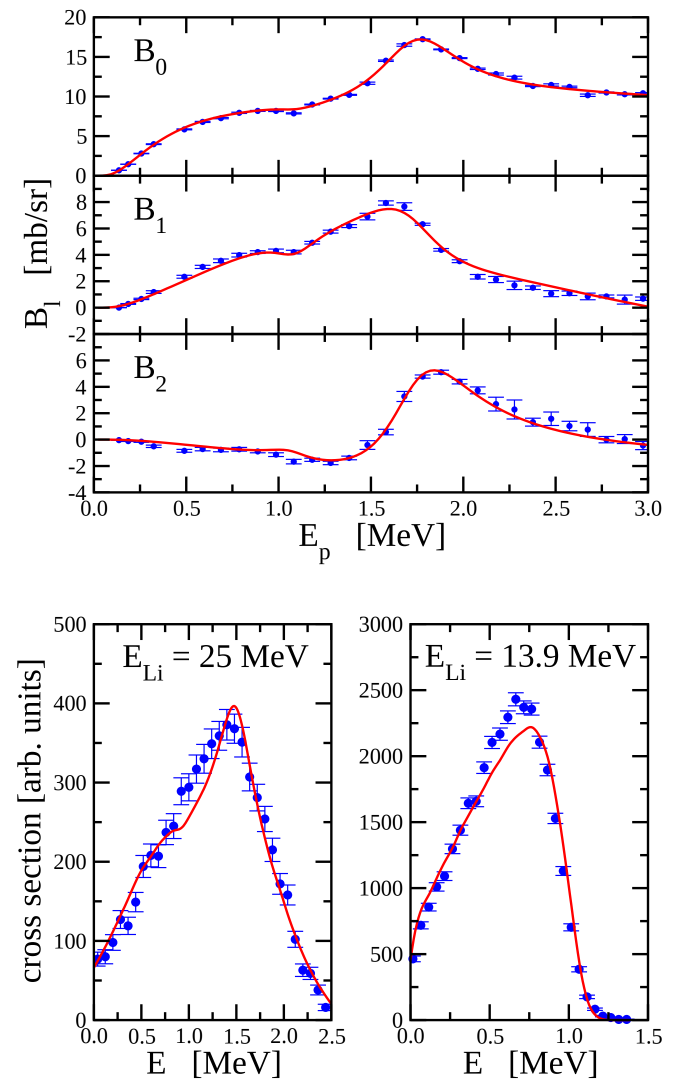
    $\Rightarrow$ simulation of THM experiment

- **THM**: ^{2}H(^{6}Li,α^3He)n
  - experiments with 13.9/25 MeV ^{6}Li beam

    (A. Tumino et al., Phys. Rev. C 67 (2003) 065803)
  - $E^{qf} = -0.24/1.35$ MeV
  - $\hbar Q_{Bb} < 30$ MeV/c
  - finite cross section at $E = 0$ MeV!



Trojan-Horse Method - Electron Screening

direct experiments:

- reduction of Coulomb barrier by electron cloud of target nucleus
- enhanced cross section at low energies

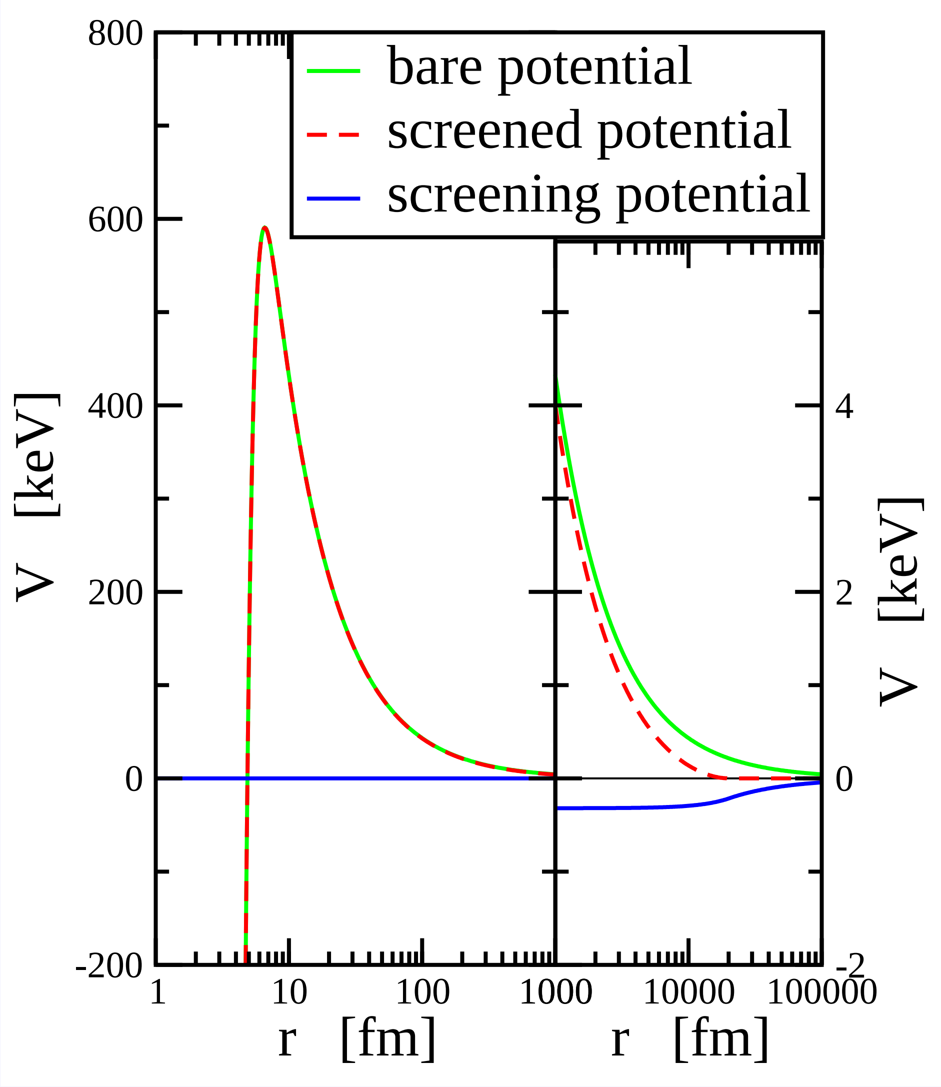
  $\sigma_{\rm exp}(E) = \sigma_{\rm bare}(E) f(E)$ with

  $f(E) = \exp(\pi \eta U_e / E)$ and

  electron screening potential energy U_e
- discrepancy between experimental observation and theoretical models, explanation?

stellar conditions:

- electron screening in plasma



Trojan-Horse Method - Example: ^{2}H(^{6}Li,α)^{4}He

- **direct reaction**: ^{2}H(^{6}Li,α)^{4}He
  - experiment with gas target
    (S. Engstler et al., Z. Phys. A 342 (1992) 471)
  - $S(0) = 17.4$ MeV b
    (corrected for electron screening)

Trojan-Horse Method - Example: ^{2}H(^{6}Li,α)^{4}He

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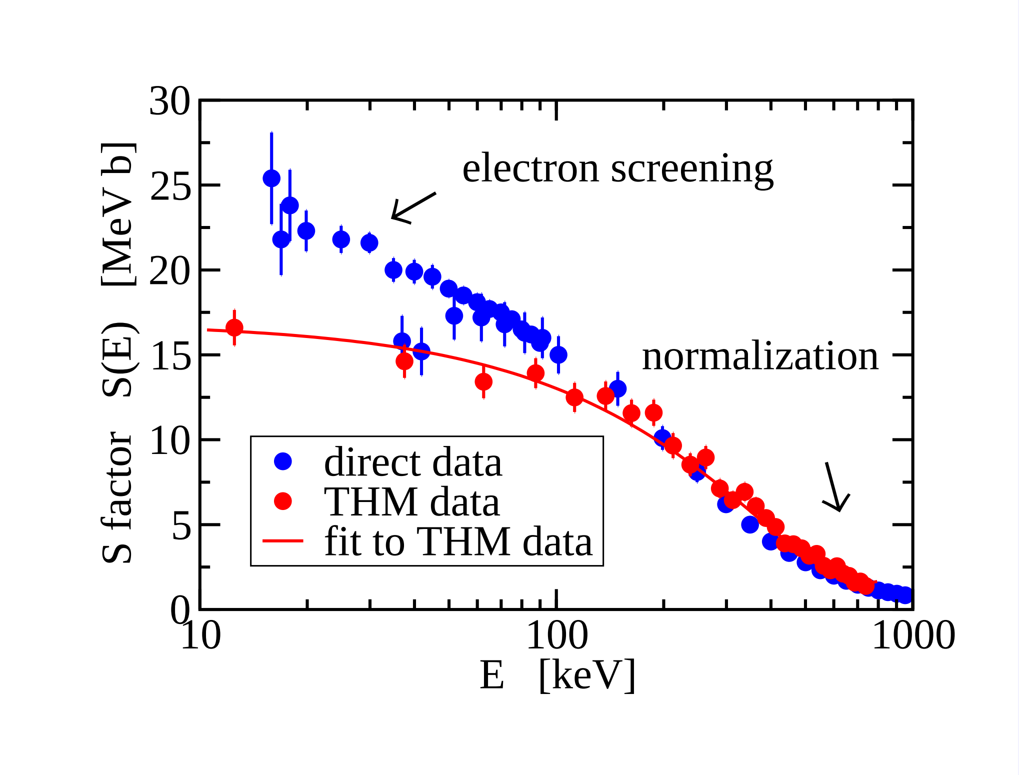
  - experiment with gas target
    (S. Engstler et al., Z. Phys. A 342 (1992) 471)
  - $S(0) = 17.4$ MeV b
    (corrected for electron screening)

- **THM**: ^{6}Li(^{6}Li,$\alpha\alpha$)^{4}He
  - experiment with 6 MeV ^{6}Li beam
    (C. Spitaleri et al., Phys. Rev. C 63 (2001) 055801;
    A. Musumarra et al., Phys. Rev. C 64 (2001) 068801)
  - $E^{qf} = 25$ keV
  - target and projectile breakup
  - $l = 0$, $\hbar Q_{Bb} < 35$ MeV/c
  - normalization to direct data
    for $E > 600$ keV
    $\Rightarrow S(0) = (16.9 \pm 0.5)$ MeV b



- **electron screening potential**:
  U_e(direct) $= (330 \pm 120)$ eV
  U_e(THM) $= (320 \pm 50)$ eV
  U_e(theory) $= 186$ eV (adiabatic limit)

Trojan-Horse Method - Extensions

- analysis only in simple theoretical approximations
  $\Rightarrow$ full DWBA calculations needed for quasi-free scattering conditions
  with consistent treatment of bound/scattering/resonant states
  (numerically very demanding)

- finite cross section at $E_{Ax} = 0 \Rightarrow$ continue to $E_{Ax} < 0$:
  investigation of subthreshold resonances

- extension to radiative capture reactions possible
  $\Rightarrow$ additional approach independent from Coulomb dissociation and ANC methods

- study elastic scattering without Coulomb contribution $\Rightarrow$ optical potentials

- application to reactions with exotic nuclei $\Rightarrow$ large cross sections

- extracted S factor not affected by electron screening
  $\Rightarrow$ determination of electron screening potential U_e by comparison to direct data
  $\Rightarrow$ consistent values for U_e, larger than adiabatic limit, challenge for theory

Summary

- **Halo nuclei**
  - unstable exotic nuclei with small binding energy
  - often well described in simple models
  - essential properties determined by asymptotics of wave functions
  - characterized by few low-energy constants
  - scaling laws
- **Indirect methods** (CD, ANC, THM)
  - provide complementary information for reactions of astrophysical interest
  - peripheral reactions
  - specific kinematical conditions
  - description with direct reaction theory
  - factorization of cross sections
  - great potential for future applications